Thm 94 (Gabriel's Theorem): Q has finitely-many indecomposibles if and only if it is a union of orientations of Dynkin diagrams of type A_n, D_n, or E_n. In this case, indecomposibles are in bijection with positive roots.

Pf uses "Reflection functors"

[Etingof et. al. §6.6-6.8] [Schiffler Ch. 3, Ch. 8]

$$A_n : V(i,j) \leftrightarrow e_i - e_j \quad (i<j)$$

D_4: Simple roots

$$\alpha_1 = e_1 - e_2, \quad \alpha_2 = e_2 - e_3, \quad \alpha_3 = e_3 - e_4, \quad \alpha_4 = e_3 + e_4$$

Other pos. roots

$$\alpha_1 + \alpha_2 = e_1 - e_3, \quad \alpha_2 + \alpha_3 = e_2 - e_4, \quad \alpha_2 + \alpha_4 = e_2 + e_4$$

$$\alpha_1 + \alpha_2 + \alpha_3 = e_1 - e_4$$

$$\alpha_1 + \alpha_2 + \alpha_4 = e_1 + e_4$$

$$\alpha_2 + \alpha_3 + \alpha_4 = e_2 + e_3$$

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = e_1 + e_3$$

$$\alpha_1 + 2\alpha_2 + \alpha_3 + \alpha_4 = e_1 + e_2$$

$$\begin{tikzcd} \mathbb{C} \arrow[r] & 0 & 0 \arrow[l] \\ & 0 \arrow[u] & \end{tikzcd} \quad \alpha_1 \qquad \begin{tikzcd} 0 \arrow[r] & 0 & \mathbb{C} \arrow[l] \\ & 0 \arrow[u] & \end{tikzcd} \quad \alpha_3 \qquad \begin{tikzcd} 0 \arrow[r] & 0 & 0 \arrow[l] \\ & \mathbb{C} \arrow[u] & \end{tikzcd} \quad \alpha_4 \qquad \begin{tikzcd} 0 \arrow[r] & \mathbb{C} & 0 \arrow[l] \\ & 0 \arrow[u] & \end{tikzcd} \quad \alpha_2$$

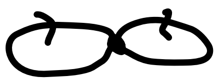
$$\begin{tikzcd} \mathbb{C} \arrow[r] & \mathbb{C} & 0 \arrow[l] \\ & 0 \arrow[u] & \end{tikzcd} \quad \alpha_1+\alpha_2 \qquad \begin{tikzcd} 0 \arrow[r] & \mathbb{C} & \mathbb{C} \arrow[l] \\ & 0 \arrow[u] & \end{tikzcd} \quad \alpha_2+\alpha_3 \qquad \begin{tikzcd} 0 \arrow[r] & \mathbb{C} & 0 \arrow[l] \\ & \mathbb{C} \arrow[u] & \end{tikzcd} \quad \alpha_2+\alpha_4$$

$$\begin{tikzcd} \mathbb{C} \arrow[r] & \mathbb{C} & \mathbb{C} \arrow[l] \\ & 0 \arrow[u] & \end{tikzcd} \quad \alpha_1+\alpha_2+\alpha_3 \qquad \begin{tikzcd} \mathbb{C} \arrow[r] & \mathbb{C} & 0 \arrow[l] \\ & \mathbb{C} \arrow[u] & \end{tikzcd} \quad \alpha_1+\alpha_2+\alpha_4 \qquad \begin{tikzcd} 0 \arrow[r] & \mathbb{C} & \mathbb{C} \arrow[l] \\ & \mathbb{C} \arrow[u] & \end{tikzcd} \quad \alpha_2+\alpha_3+\alpha_4$$

$$\begin{tikzcd} \mathbb{C} \arrow[r] & \mathbb{C} & \mathbb{C} \arrow[l] \\ & \mathbb{C} \arrow[u] & \end{tikzcd} \quad \alpha_1+\alpha_2+\alpha_3+\alpha_4 \qquad \begin{tikzcd} \mathbb{C} \arrow[r] & \mathbb{C}^2 & \mathbb{C} \arrow[l] \\ & \mathbb{C} \arrow[u] & \end{tikzcd} \quad \alpha_1+2\alpha_2+\alpha_3+\alpha_4$$

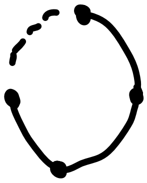
Drozd: Non-finite-type quivers are either <u>tame</u> or <u>wild</u>

- Tame: Finitely-many one-parameter families of indecomps. plus finitely-many exceptions
- Wild: Contains the problem of classifying repns of

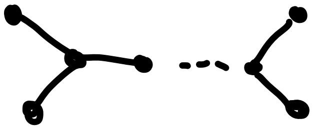


all wild problems contain all other wild problems

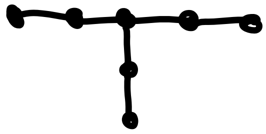
Tame quivers are orientations of these graphs:



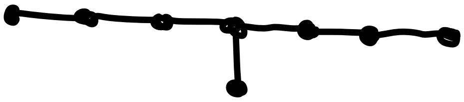
$\tilde{A}_n$



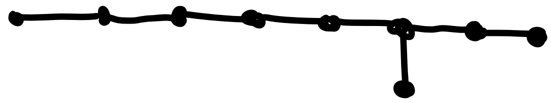
$\tilde{D}_n$



$\tilde{E}_6$



$\tilde{E}_7$



$\tilde{E}_8$

Def 95: a) The <u>path algebra</u> of a quiver Q is the $\mathbb{C}$-alg. $\mathbb{C}Q$ spanned by all paths in Q, including the "lazy paths" $\{e_v, v \in Q_0\}$, with relations

$$p \cdot q = \{\underbrace{p \circ q}_{\text{concatenation}} \text{ if } q \text{ ends where } p \text{ begins, else } 0\}$$

The category of f.d. left $\mathbb{C}Q$-modules is equiv. to the category of f.d. Q-repns. via

$$M \mapsto V \quad \text{where } V_v = e_v M$$

$$v \xrightarrow{\alpha} w \qquad V_\alpha : e_v M \to e_w M$$
$$e_v m \mapsto \alpha m$$

$$V \mapsto M = \bigoplus_{v \in Q_0} V_v \quad \mathbb{C}Q\text{-action by the maps } V_\alpha.$$

Ex: $Q = \begin{tikzcd} \underset{1}{\bullet} \arrow[r, "\alpha"] & \underset{2}{\bullet} \arrow[r, "\beta"] & \underset{3}{\bullet} \end{tikzcd}$

$$\mathbb{C}Q = \operatorname{span}\{e_1, e_2, e_3, \alpha, \beta, \beta\alpha\} \cong \begin{bmatrix} \mathbb{C} & 0 & 0 \\ \mathbb{C} & \mathbb{C} & 0 \\ \mathbb{C} & \mathbb{C} & \mathbb{C} \end{bmatrix} \qquad \begin{bmatrix} e_1 & & \\ \alpha & e_2 & \\ \beta\alpha & \beta & e_3 \end{bmatrix}$$

b) Let $R_Q \subseteq \mathbb{C}Q$ be the two-sided ideal generated by all arrows. A two-sided ideal $I \subseteq \mathbb{C}Q$ is <u>admissible</u> if $R_Q^m \subseteq I \subseteq R_Q^2$ for some $m \geq 2$. A <u>bound quiver algebra</u> is a quotient $\mathbb{C}Q/I$ where I is admissible.

e.g. $\begin{tikzcd} \underset{1}{\bullet} \arrow[r, "\alpha"] & \underset{2}{\bullet} \arrow[r, "\beta"] & \underset{3}{\bullet} \end{tikzcd}$ $\qquad I = (\beta\alpha)$ admissible

$$\mathbb{C}Q/I = \operatorname{span}\{e_1, e_2, e_3, \alpha, \beta\}$$

Quotient loses indecomp. $\mathbb{C} \xrightarrow{1} \mathbb{C} \xrightarrow{1} \mathbb{C}$ since $\beta\alpha = 0$

Thm 96: Let A be a f.d. $\mathbb{C}$-alg. There exists a bound quiver algebra $\mathbb{C}Q/I$ s.t there is an equivalence of categories

$$A\text{-mod} \cong \mathbb{C}Q/I\text{-mod}$$

Ex: a) $A = \mathbb{C}[G]$, G: finite gp. $V_1, \ldots, V_k$ irreps of G

$Q = \overset{1}{\bullet} \quad \overset{2}{\bullet} \quad \cdots \quad \overset{k}{\bullet}$ (no arrows) $\qquad I = 0$

Q-reps are direct sums of $S(1), \ldots, S(k)$

b) $A = \mathbb{C}[x]/(x^n)$

One simple A-module: $V = \mathbb{C}$, where x acts by 0.

Indecomps: $M_k = \mathbb{C}[x]/(x^k)$, $1 \le k \le n$

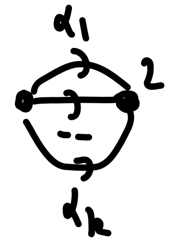
(Jordan blocks of size $\le n$ w/ evalue 0)

$Q = \bullet \circlearrowleft_{\alpha}$ $\quad I = (\alpha^n)$

General setting (Gabriel quiver):

Q has one vertex v_S for each simple A-mod S.

Q has $\dim \mathrm{Ext}^1(T, S)$ arrows from v_S to v_T.

Ex: In the quiver , we have k exact sequences

$$0 \to S(2) \hookrightarrow P_i(1) \twoheadrightarrow S(1) \to 0$$