Math 418, Spring 2025 – Practice Problems 2

13.2.6 *Prove directly from the definitions that the field $F(a_1, \ldots, a_n)$ is the composite of the fields $F(a_1), F(a_2), \ldots, F(a_n)$.*

13.3.1 *Prove that it is impossible to construct the regular 9-gon.*

13.4.4 *Determine the splitting field and its degree over $\mathbb{Q}$ for $f(x) = x^6 - 4$.*

13.5.2 *Find all irreducible polynomials of degrees 1, 2 and 4 over $\mathbb{F}_2$ and prove that their product is $x^{16} - x$.*

13.5.4 *Let $a > 1$ be an integer. Prove for any positive integers n, d that d divides n if and only if $a^d - 1$ divides $a^n - 1$. Conclude in particular that $\mathbb{F}_{p^d} \subseteq \mathbb{F}_{p^n}$ if and only if d divides n.*

13.6.6 *Prove that for n odd, $n > 1$ that $\Phi_{2n}(x) = \Phi_n(-x)$*

13.6.10 *Let ϕ denote the Frobenius map $\mathbb{F}_{p^n}\cdot$ Prove that ϕ gives an automorphism of order n*

14.1.1 (a) *Show that if the field K is generated over F by the elements $a_1, ..., a_n$ then an automorphism a of K fixing F is uniquely determined by $\sigma(a_1), \ldots, \sigma(a_n)$. In particular, show that an automorphism fixes K if and only if it fixes a set of generators for K.*

(b) *Let $G \leq Gal(K/F)$ be a subgroup of the Galois group of the extension K/F and suppose $\sigma_1, \ldots, \sigma_k$ are generators for G. Show that the subfield E of K containing F is fixed by G if and only if it is fixed by the generators $\sigma_1, \ldots, \sigma_k$.*

14.1.9 *Determine the fixed field of the automorphism $\phi : t \mapsto t + 1$ of $k(t)$*

14.1.10 *Let K be an extension of the field F. Let $\phi : K \to K'$ be an isomorphism of K with a field K' which maps F to the subfield F' of K'. Prove that the map $\sigma \mapsto \phi\sigma\phi^{-1}$ defines a group isomorphism $Aut(K/F) \to Aut(K'/F)$.*