Math 418, Spring 2025 – Homework 4

Due: Wednesday, February 26th, at 9:00am via Gradescope.

Instructions: Students should complete and submit all problems. Textbook problems are from Dummit and Foote, *Abstract Algebra, 3rd Edition*. All assertions require proof, unless otherwise stated. Typesetting your homework using LaTeX is recommended, and will gain you 1 bonus point per assignment.

1. **Dummit and Foote #13.2.1:** *Let $\mathbb{F}$ be a finite field of characteristic p. Prove that $|\mathbb{F}| = p^n$ for some positive integer n.*

2. **Dummit and Foote #13.2.4:** *Determine the degree over $\mathbb{Q}$ of $2 + \sqrt{3}$ and of $1 + \sqrt[3]{2} + \sqrt[3]{4}$.*

3. **Dummit and Foote #13.2.5:** *Let $F = \mathbb{Q}(i)$. Prove that $x^3 - 2$ and $x^3 - 3$ are irreducible over F.*

4. **Dummit and Foote #13.2.7:** *Prove that $\mathbb{Q}(\sqrt{2}+\sqrt{3}) = \mathbb{Q}(\sqrt{2}, \sqrt{3})$. Conclude that $[\mathbb{Q}(\sqrt{2}+\sqrt{3}) : \mathbb{Q}] = 4$. Find an irreducible polynomial satisfied by $\sqrt{2}+\sqrt{3}$.*

5. **Dummit and Foote #13.3.2:** *Prove that Archimedes' construction actually trisects the angle θ. (See the book for the construction).*

6. **Dummit and Foote #13.3.4:** *The construction of the regular 7-gon amounts to the constructibility of $\cos(2\pi/7)$. We shall see later (Section 14.5 and Exercise 2 of Section 14.7) that $\alpha = 2\cos(2\pi/7)$ satisfies the equation $p(x) = x^3 + x^2 - 2x - 1 = 0$. Use this to prove that the regular 7-gon is not constructible by straightedge and compass.*

7. **Dummit and Foote #13.3.5:** *Use the fact that $\alpha = 2\cos(2\pi/5)$ satisfies the equation $x^2 + x - 1 = 0$ to conclude that the regular $5-$gon is constructible by straightedge and compass.*