

Math 418, Spring 2024 – Homework 5

Due: Wednesday, February 28th, at 9:00am via Gradescope.

Instructions: Students should complete and submit all problems. Textbook problems are from Dummit and Foote, *Abstract Algebra, 3rd Edition*. All assertions require proof, unless otherwise stated. Typesetting your homework using LaTeX is recommended, and will gain you 2 bonus points per assignment.

1. **Dummit and Foote #13.4.1:** *Determine the splitting field and its degree over \mathbb{Q} for $f(x) = x^4 - 2$.*
2. **Dummit and Foote #13.4.2:** *Determine the splitting field and its degree over \mathbb{Q} for $f(x) = x^4 + 2$.*
3. **Dummit and Foote #13.4.3:** *Determine the splitting field and its degree over \mathbb{Q} for $f(x) = x^4 + x^2 + 1$.*
4. **Dummit and Foote #13.4.6:** *Let K_1 and K_2 be finite extensions of F contained in the field K , and assume both are splitting fields over F .*
 - a. *Prove that their composite K_1K_2 is a splitting field over F .*
 - b. *Prove that $K_1 \cap K_2$ is a splitting field over F .*

[Note: you may assume the result of the prior Dummit & Foote exercise: K is a splitting field for F if and only if every irreducible polynomial in $F[x]$ with a root in K splits completely in $K[x]$]