Math 418, Spring 2024 – Homework 4

Due: Wednesday, February 21st, at 9:00am via Gradescope.

Instructions: Students should complete and submit all problems. Textbook problems are from Dummit and Foote, *Abstract Algebra, 3rd Edition.* All assertions require proof, unless otherwise stated. Typesetting your homework using LaTeX is recommended, and will gain you 2 bonus points per assignment.

1. **Dummit and Foote #13.2.1:** *Let* $\mathbb{F}$ *be a finite field of characteristic* p*. Prove that* $|\mathbb{F}| = p^n$ *for some positive integer* n*.*

2. **Dummit and Foote #13.2.4:** *Determine the degree over* $\mathbb{Q}$ *of* $2+\sqrt{3}$ *and of* $1+\sqrt[3]{2}+\sqrt[3]{4}$*.*

3. **Dummit and Foote #13.2.5:** *Let* $F = \mathbb{Q}(i)$*. Prove that* x^3-2 *and* x^3-3 *are irreducible over* F*.*

4. **Dummit and Foote #13.2.7:** *Prove that* $\mathbb{Q}(\sqrt{2}+\sqrt{3}) = \mathbb{Q}(\sqrt{2},\sqrt{3})$*. Conclude that* $[\mathbb{Q}(\sqrt{2}+\sqrt{3}) : \mathbb{Q}] = 4$*. Find an irreducible polynomial satisfied by* $\sqrt{2}+\sqrt{3}$*.*

5. **Dummit and Foote #13.3.2:** *Prove that Archimedes' construction actually trisects the angle* θ*. (See the book for the construction).*

6. **Dummit and Foote #13.3.4:** *The construction of the regular 7-gon amounts to the constructibility of* $\cos(2\pi/7)$*. We shall see later (Section 14.5 and Exercise 2 of Section 14. 7) that* $\alpha = 2\cos(2\pi/7)$ *satisfies the equation* $p(x) = x^3+x^2-2x-1 = 0$*. Use this to prove that the regular 7-gon is not constructible by straightedge and compass.*

7. **Dummit and Foote #13.3.5:** *Use the fact that* $\alpha = 2\cos(2\pi/5)$ *satisfies the equation* $x^2+x-1=0$ *to conclude that the regular* $5-$*gon is constructible by straightedge and compass.*