Announcements:

- Today, course evals, quiz 4, Four-Color Theorem
- No problem session tomorrow
  (go to the Tondeur lecture instead!)
- Exam review: Wed., plus review session & office hours next week
- Final exam: Thurs 12/14, 8:00-11:00am, 132 Bevier Hall

Start of class: course evals (ices.citl.illinois.edu)
(optional, but appreciated; or do it on own time)

Quiz will start at <u>10:10</u>! (I'll be back @ 10:08)

Last time: gave Kempe's "proof" of the 4-color theorem.
But, there was a subtle flaw!

Recall:

G_{ij} = induced subgraph of $G \setminus v$ consisting of vertices of color i or j

P_{ij} = path from v_i to v_j in G_{ij} (if it exists/makes sense)

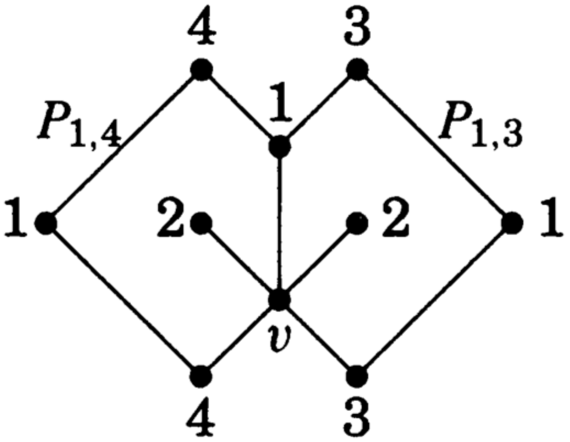
H = component of $G_{2,4}$ containing v_2

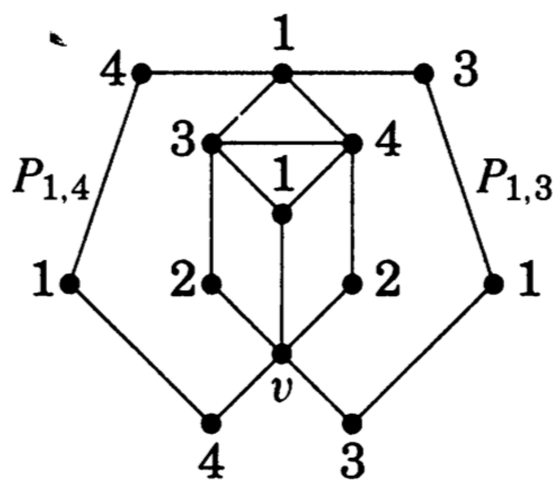
H' = component of $G_{2,3}$ containing v_5

Kempe obtained a proper 4-coloring of G by

- Swapping colors $2 \leftrightarrow 4$ on H
- Swapping colors $2 \leftrightarrow 3$ on H'
- Color v color 2

However, this doesn't work for the possibility on the right:





History of four-color theorem

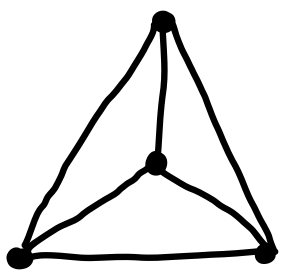
1852: problem first stated (Francis Guthrie)

1878: Cayley announces problem to London Mathematical Society

1879: Kempe publishes "proof"

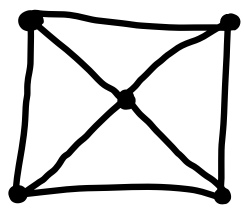
1890: Heawood finds error in Kempe's proof, proves 5-color thm.

Kempe gave us the following set of unavoidable configurations:



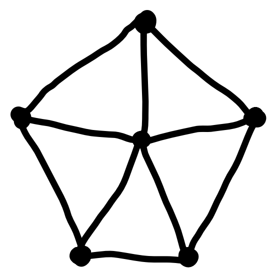
•3

reducible



•4

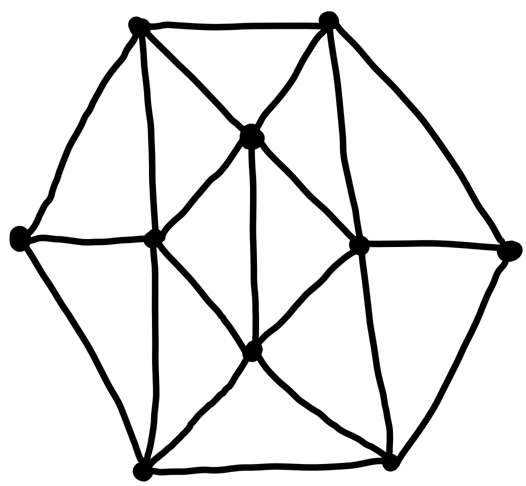
reducible



•5

not obviously reducible

Since we can't reduce •5, we replace it with **many** more configurations: $\exists$ unavoidable set containing •3, •4, and hundreds/thousands of configurations, e.g. ⟶



1910s – 1960s: More configurations were found

Theorem [Appel & Haken (with Koch), 1976, @ UIUC!]:
Every planar graph is 4-colorable.

Method of proof:

- Found an unavoidable set of 1936 configurations
- For each configuration, check that each proper 4-coloring the ring leads to a way to color the interior
- Used 1200 hours of computer time to check all these cases

Very controversial at first, but has come to be accepted

1981: Error found (Schmidt)

1989: Appel & Haken published book with detailed explanation and fixing errors

1996: Simplified proof using only 633 configurations

2005: Gonthier and Werner "formalized" a proof of the Four-Color Theorem using the Coq proof assistant