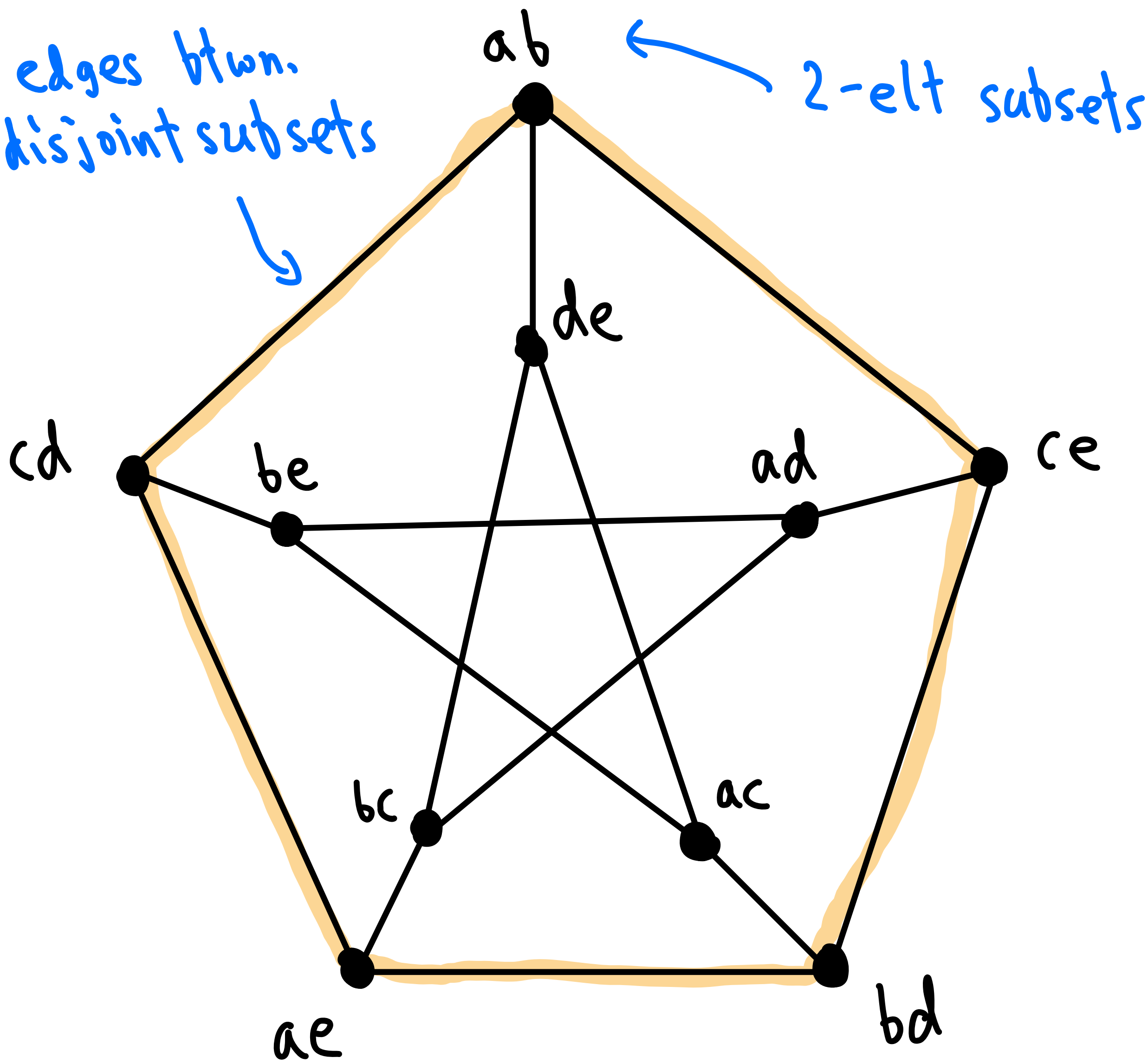
Announcements:

- HW 1 due Wed. 9am via Gradescope
  Course code: 57YPR7

don't be late!

- Problem session tomorrow 4pm-5:30pm Henry Admin 156

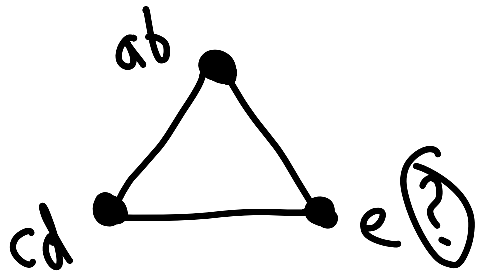
Petersen graph: $S = \{a, b, c, d, e\}$



Def 1.1.39: The <u>girth</u> of a graph is the length of its shortest cycle (no cyles: girth = ∞)

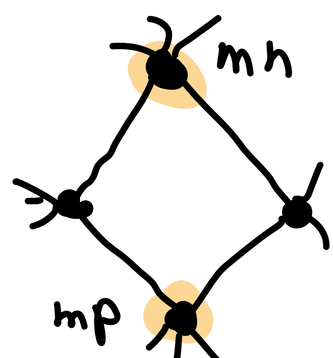
Cor 1.1.40: The Petersen graph G has girth 5.

Pf: G is simple, so it has no 1-cycles or 2-cycles. If G contains a 3-cycle,



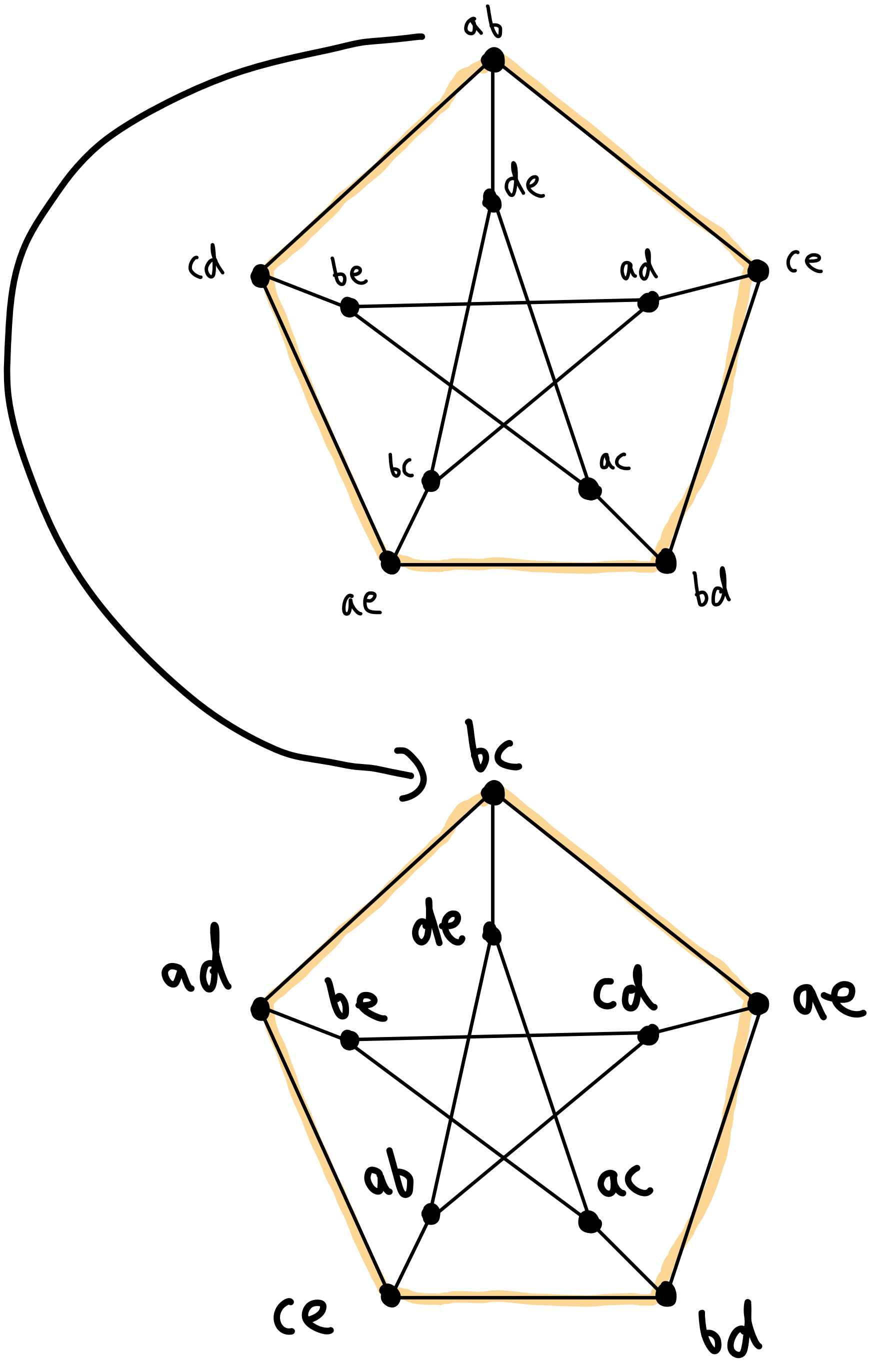
this means that S has 3 pairwise disjoint subsets of size 2, which mean $|S| \geq 6$, a contradiction.

If G has a 4-cycle, consider two nonadjacent vertices in this cycle. The corresp. subsets are not disjoint or equal, so they



are of the form $\{m,n\}$ and $\{m,p\}$, $m,n,p \in S$. However, $S \setminus \{m,n,p\}$ has order 2, so $S \setminus \{m,n,p\}$ is the only 2-elt subset of S disjoint to $\{m,n\}$ and $\{m,p\}$; therefore, there is precisely one vertex adjacent to those two, which contradicts the assumption that they lie in a 4-cycle.

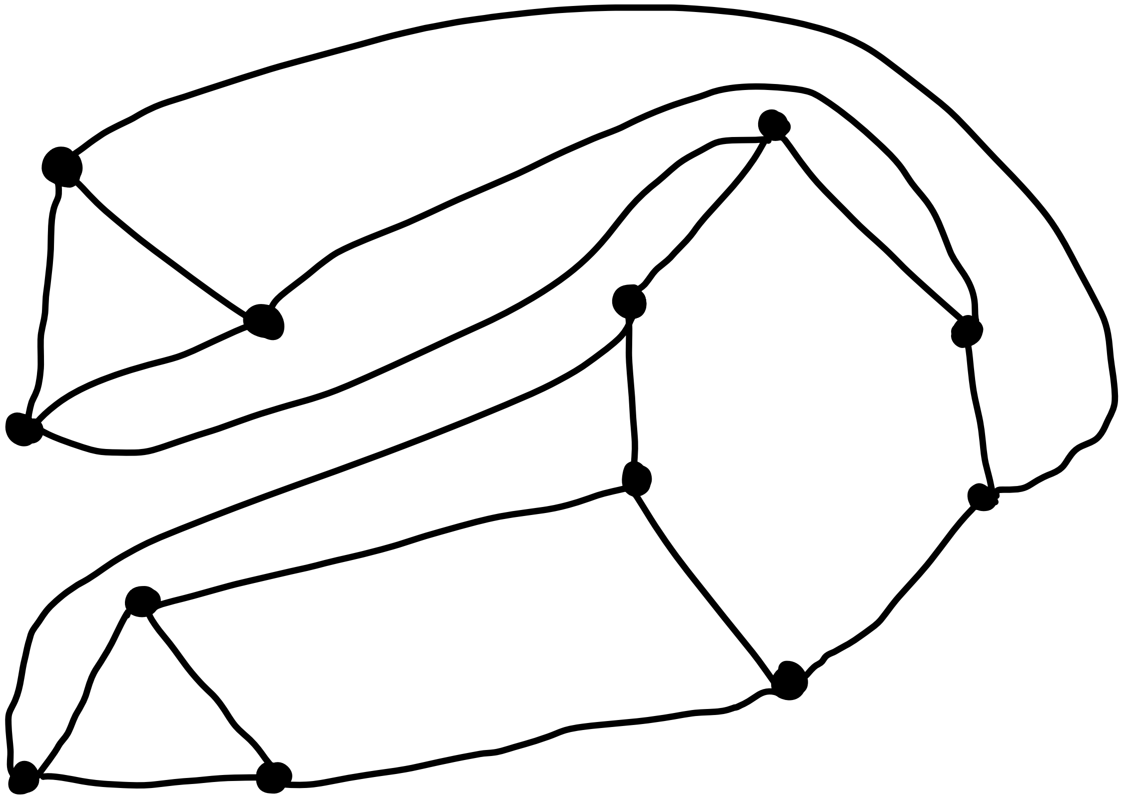
Finally, the vertices corresponding to the subsets $\{ab\} - \{cd\} - \{ae\} - \{bd\} - \{ce\} - \{ab\}$ form a 5-cycle, so the Petersen graph has girth 5.



$f(ab) = bc$

$f(cd) = ad$

$\vdots$



Def 1.1.41:

a) An <u>automorphism</u> is an isomorphism from a graph to itself [These form a group]

b) A graph G is vertex-transitive if for every pair of vertices $u, v \in V(G)$, there is an automorphism of G mapping u to v

Remark: The Petersen graph is vertex transitive

§1.2: Paths, Cycles, & Trails

Def 1.2.2:

a) A walk is a list

$$v_0, e_1, v_1, e_2, \dots, e_k, v_k, \quad e_i \in E(G), v_i \in V(G)$$

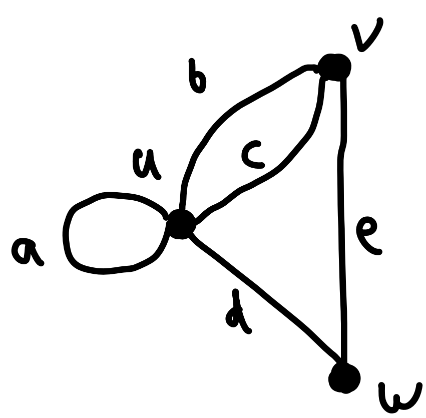
such that e_i has endpoints v_{i-1} and v_i

b) A trail is a walk w/ no repeated edges

c) (Recall) A path is a walk w/ no repeated vertices (or edges)

A walk is closed if $v_0 = v_k$

Class activity: Walk, trail, path, or none? Closed?
(W) (T) (P) (N) (C)



a) u, a, u, c, v, b, u, d, w   WT

b) w, d, v, a, u, c, b   N

c) u, b, v, e, w   WTP

d) u, b, v, e, w, d, u   WT   C

e) u, c, v, e, w, e, v, b, u   W   C

Note: If the graph is simple, we just list vertices start ↘ ↙ end

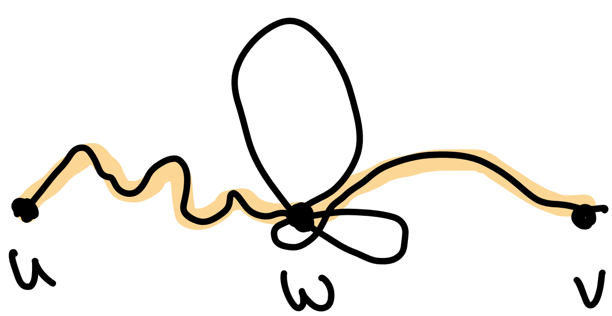
Lemma 1.2.5: Every u,v-walk contains a u,v-path

Pf: Induction on the length of the walk W

Base case: Length $= 0$ (i.e. no edges)

Then $u = v$ and W is just the vertex u, so no repeated vertices, so W is a u,v-path
$\qquad\qquad\qquad\qquad\shortparallel$
$\qquad\qquad\qquad\quad u,u$-path

Induction step: Length ≥ 1. If W has no repeated vertex, it is a u,v-path. Otherwise, assume that all shorter u,v-walks contain a u,v-path. Suppose w is a repeated vertex;



delete everything btwn the first and last occurence of w (inc. last occurence, but not the first). The result is a shorter u,v-walk W' contained in W; by the inductive hypothesis, $\exists$ a u,v-path $P \subseteq W' \subseteq W$. $\square$