Announcement:

Midterm 3 tonight!

7:00pm - 8:30pm in 217 Noyes Lab. (ref. sheet allowed)

Be early!

Exam covers through Chapter 5 (focus on Ch. 4, 5)

Most focus: topics that appeared in lecture or homework

Some focus : topics in relevant subsections of textbook

Low/no focus: topics in subsections we didn't cover at all

Partial topics list: (plus, see first two lists)

Vertex/edge connectivity:

Def'ns

Whitney's Thm.

Different characterizations of 2-connectivity and 2-edge-connectivity

Digraph vertex/edge connectivity

Menger's Theorem (4 versions)

Max-flow, min-cut theorem

- Def'ns
- Theorem itself
- Ford-Fulkerson algorithm

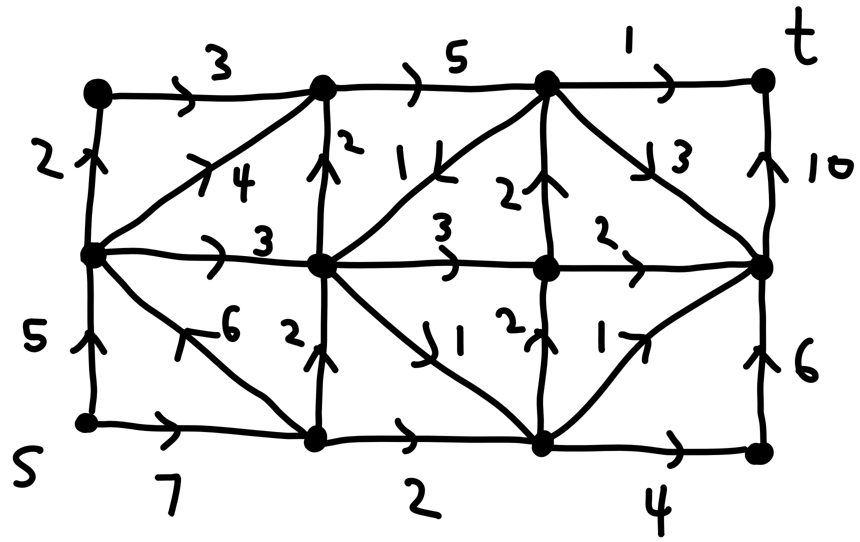
Connections between: flows, cuts, (edge)-disjoint paths, matchings, indep. sets, vertex/edge covers, etc.

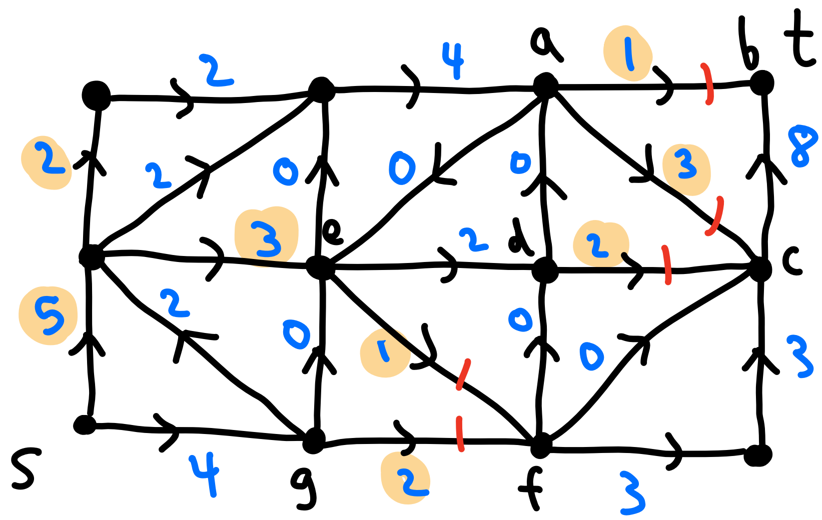
Vertex coloring

- Def'ns (e.g. Chromatic number, k-criticality)
- "Easy bounds", and more difficult ones (e.g. Brooks' Thm.)
- Greedy coloring
  - Algorithm
  - Consequences
- Mycielski's construction and theorem
- Chromatic polynomial
  - Values/how to compute for small graphs
  - Deletion-contraction recurrence

Examples:

1) Find and prove a minimum capacity source-sink cut!





Edge cut capacity: 9

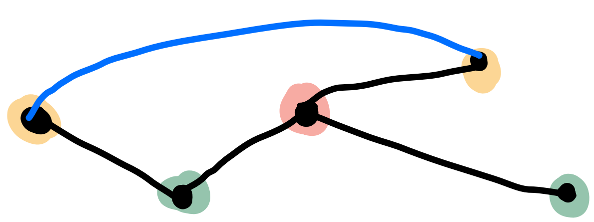
Flow value: 9

By the max-flow, min-cut Thm., this is a maximum flow and minimum cut. □

Edge cut: {ab, ac, dc, ef, gf}

2) Prove that the number of proper k-colorings of a conn. simple graph G is $< k(k-1)^{n-1}$ if $k \geq 3$ and G is not a tree. $\boxed{\chi(G;k) \leq \chi(T;k) = k(k-1)^{n-1}}$

Pf: Let T be a spanning tree of G. Any proper coloring of G is a proper coloring of T, so $\chi(G;k) \leq k(k-1)^{n-1}$, and the result will follow if $\exists$ a proper k-coloring of T that isn't a proper k-coloring of G. Since G is not a tree, let $e \in E(G) \setminus E(T)$. Since T is a tree, it is bipartite, so 2-colorable. Take such a coloring, and change the endpoints of e to a third color. Since $k \geq 3$, this is a proper k-coloring of T, but not of G, so $\chi(G;k) < k(k-1)^{n-1}$. $\square$



3) Let G be a simple graph s.t. $\overline{G}$ is bipartite. Show that $\chi(G) = \omega(G)$.

Pf: We know (Prop 5.1.7) that $\chi(G) \geq \omega(G)$. If $\overline{G}$ has any iso. vertex, it is adjacent to every vertex in G, increasing both the clique number and chromatic number by 1. Thus, we can assume $\overline{G}$ has no iso. verts.

Let $T \subseteq V(G)$ be a maximum clique in G i.e. a maximum indep. set in $\overline{G}$, and if $\overline{G}$ has partite sets X and Y, let

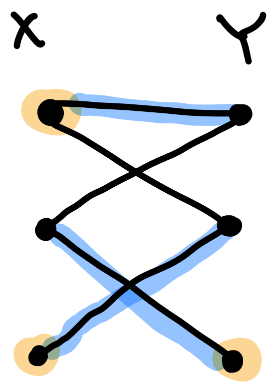
$$A = X \cap T, \quad B = Y \cap T.$$

We show that $\exists$ matching M of $\overline{G}$ s.t.

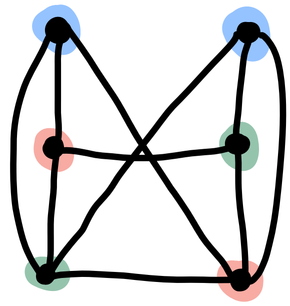
a) one endpoint for every edge in M is in T

b) M saturates $V(G) \setminus T$.

In this case, we have partitioned $V(G)$ into $|T|$ indep. sets in G (of size 1 or 2)



$\overline{G}$ T



G

Let H_1 be the induced subgraph of $\overline{G}$ w/ vertex set $X \setminus A \cup Y$ and H_2 be the induced subgraph of $\overline{G}$ w/ vertex set $X \cup Y \setminus B$. Both H_1 and H_2 are bipartite, and if $S \subseteq X \setminus A$, then

$(B \setminus N(S)) \cup S \cup A$ is an indep. set. of

size $|A| + |B| + |S| - |N(S)|$. Since the maximum size of an indep. set in $\overline{G}$ is $|A| + |B|$, we must have $|S| \leq |N(S)|$, so Hall's condition is satisfied, and there exists a matching in H_1 that saturates $X \setminus A$. Similarly, there exists a matching in H_2 that saturates $Y \setminus B$. The union of these two matchings is the desired matching in $\overline{G}$. $\square$