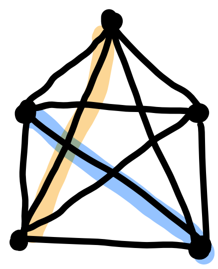
Announcements:

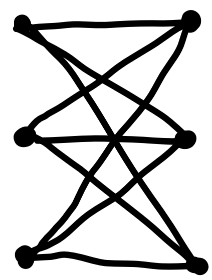
Midterm 3: Wed. 7:00-8:30pm Noyes 217

Covers through Chapter 5

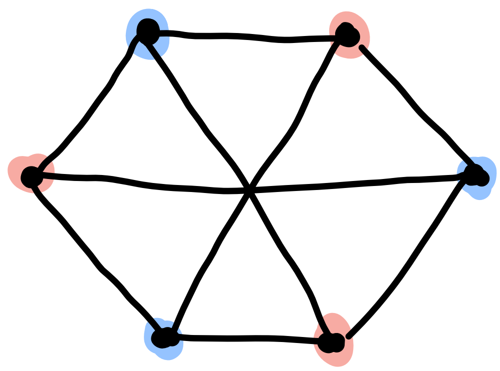
Final homework (HW9) will be due Wed. 11/29

Prop 6.1.2: K_5 and $K_{3,3}$ are not planar

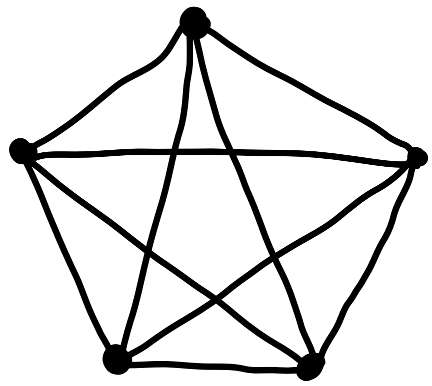




Pf: Let C be a spanning cycle. If G, K_5 or $K_{3,3}$, has a planar embedding, every other edge goes inside or outside C. When two such edges have alternating endpoints, they "conflict", and one must go outside and the other inside. However, $K_{3,3}$ consists of C and 3 pairwise conflicting edges, while K_5 consists of C and 5 more edges, any 3 of which contain a conflict $\square$

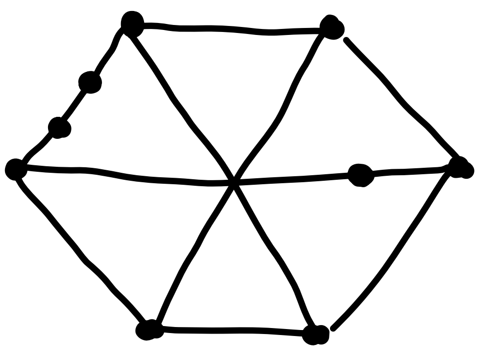


$K_{3,3}$



K_5

Def: A <u>subdivision</u> of a graph G is a graph G' obtained by repeated subdivisions of edges



Kuratowski's Theorem (6.2.2): Let G be a graph.

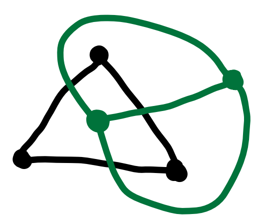
G is planar $\iff$ G does not have a subgraph isomorphic to a subdivision of K_5 or $K_{3,3}$.

Pf: $\Rightarrow$ follows from Prop 6.1.2 since if G is planar then any subgraph is planar and since subdivisions don't affect planarity.

$\Leftarrow$ is difficult, and we probably won't have time for it (see West Section 6.2)

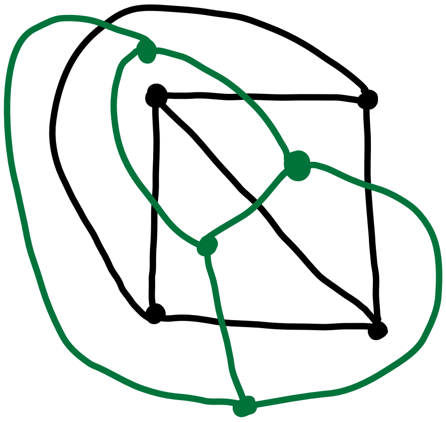
For any plane graph G (loops, mult. edges ok!), there is a nice relationship between vertices, edges, and faces.

Def 6.1.7: Let G be a plane graph. The <u>dual graph</u> G^* of G is a plane graph whose vertices corresp. to the faces of G. For each edge e in G, we create an edge in G^* crossing e, with endpoints at the vertices of G^* corresponding to the faces of G bounding e.



Class activity: Find the dual graphs, and count the vertices, edges, and faces of G and G^*.

a)

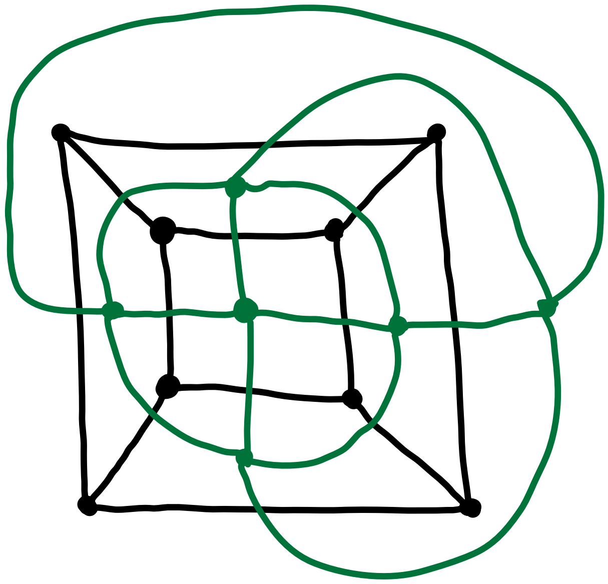


v, e, f

4, 6, 4

4, 6, 4

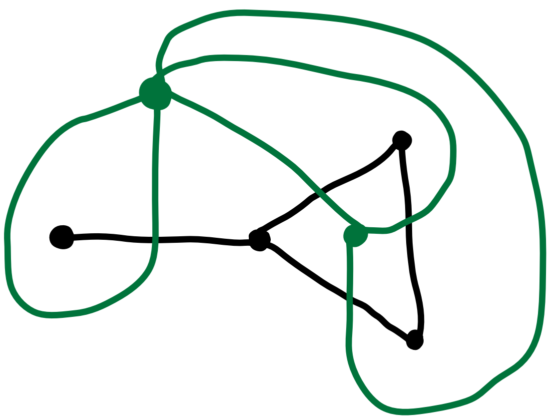
b)



8, 12, 6

6, 12, 8

c)

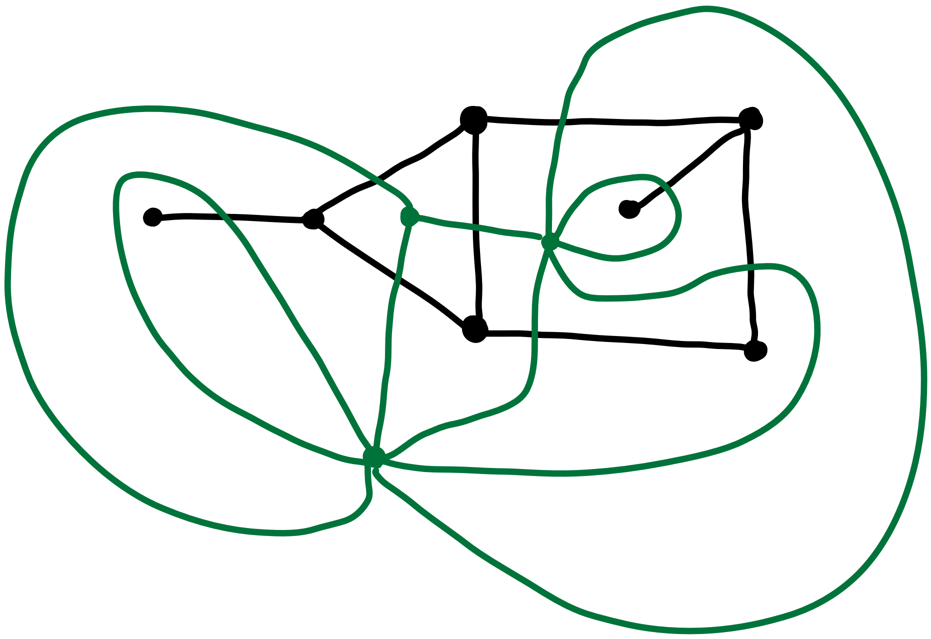


4, 4, 2

2, 4, 4

Class activity: Same thing!

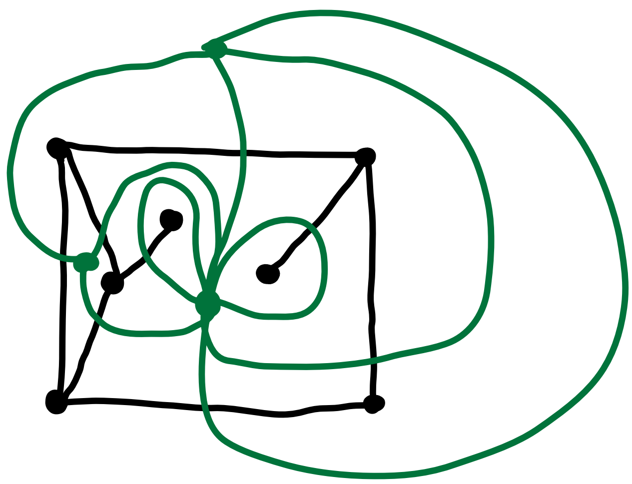
a)



7, 8, 3

3, 8, 7

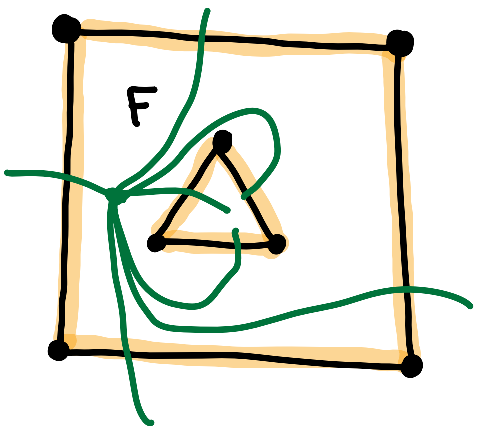
b)



7, 8, 3

3, 8, 7

Def 6.1.11: The length $\ell(F)$ of a face F in a plane graph G is the total length of the closed walk(s) in G bounding F.



$\ell(F) = 7$

Prop 6.1.13: Let G be a plane graph.

a) Let F be a face of G, and let $v \in V(G^*)$ be the corresponding vertex in G^*. Then, $\ell(F) = d(v)$.

b) If $F_1, \ldots, F_k$ are the faces of G, then

$$2e(G) = \sum_{i=1}^{k} \ell(F_i).$$

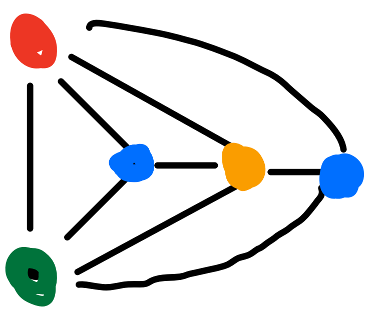
c) The chromatic number $\chi(G^*)$ is the smallest number of colors needed to color the faces of G such that no faces which share a boundary edge have the same color.

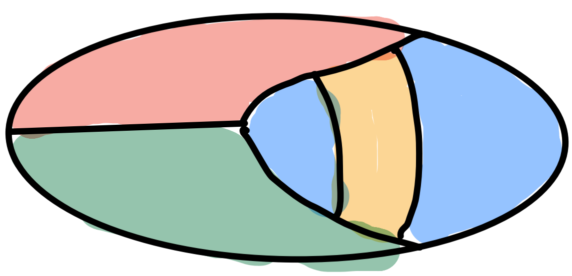
Pf: a) This follow from the construction of G^* since we draw an edge incident to v for every edge on the boundary of F.

b) Let $v_i \in V(G^*)$ correspond to F_i. Using a) and the degree sum formula, we have

$$2e(G) = 2e(G^*) = \sum_{i=1}^{k} d(v_i) = \sum_{i=1}^{k} l(F_i)$$

c) This is the case since every proper coloring of G corresponds to such a coloring of the faces of G^*





$\square$

Thm 6.1.14: Let G be a connected graph.

Let $D \subseteq E(G)$, and let $D^* \in E(G^*)$ be the corresponding edges in G^*. Then,

D is the edge set of a cycle $\iff$ D^* is a minimal edge cut.