Announcements:

Quiz today!

Midterm 3: Next Wed. 11/15 7:00-8:30pm Noyes 217

Recall: The chromatic polynomial of G is

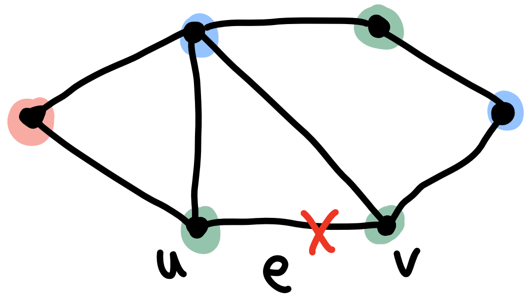
$\chi(G;k) :=$ number of proper k-colorings of G

There is a method to compute $\chi(G;k)$ recursively using deletion-contraction, allowing for a computation of $\chi(G;k)$, and thus $\chi(G)$, for any (individual) graph G.

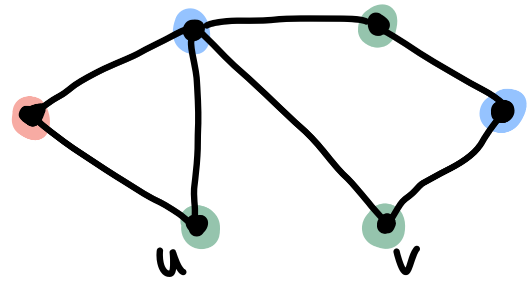
Thm 5.3.6: Let G be a simple graph and $e \in E(G)$. Then,

$$\chi(G;k) = \chi(G \setminus e;k) - \chi(G \cdot e;k)$$

Pf: Every proper coloring of G is a proper coloring of $G \setminus e$, and a proper coloring of $G \setminus e$ is a proper coloring of G iff it give distinct colors to the endpoints u and v of e



G



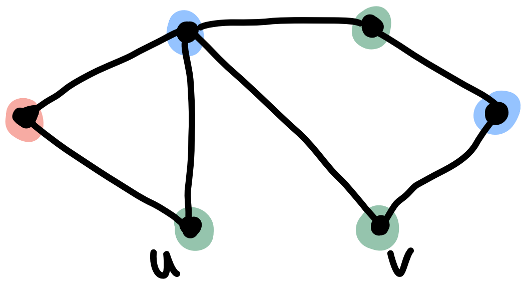
$G \setminus e$

So

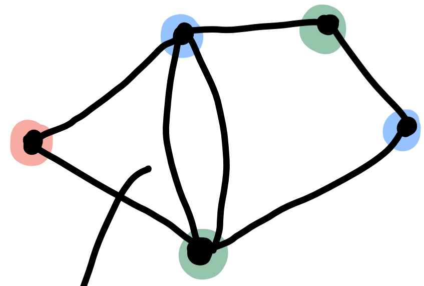
$$\chi(G;k) = \chi(G \setminus e;k) - \left| \left\{ \begin{array}{l} \text{proper } k\text{-colorings of } G \setminus e \\ \text{where } u,v \text{ have same color} \end{array} \right\} \right|$$

and

$$\left\{ \begin{array}{c} \text{proper } k\text{-colorings} \\ \text{of } G \setminus e \text{ where} \\ u,v \text{ have same color} \end{array} \right\} \leftrightarrow \left\{ \begin{array}{c} \text{proper colorings} \\ \text{of } G \cdot e \end{array} \right\}$$



$G \setminus e$



$G \cdot e$

can remove mult. edges

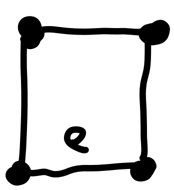
$\square$

Example 5.3.7:

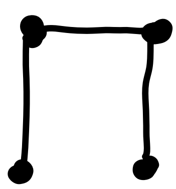
a) Let $G = C_4$, $e \in E(G)$ any edge

Then $G \setminus e \cong P_4$ $\quad \chi(P_4; k) = k(k-1)^3$

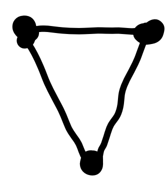
$G \cdot e \cong K_3$ $\quad \chi(K_3; k) = k(k-1)(k-2)$



$G = C_4$



$G \setminus e = P_4$



$G \cdot e = C_3 = K_3$

So,

$$\chi(G;k) = \chi(P_4;k) - \chi(K_3;k) = k(k-1)(k^2-3k+3)$$

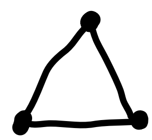
Chapter 6: Planar Graphs

Goal: Find possible values of $\chi(G)$ for planar graphs G

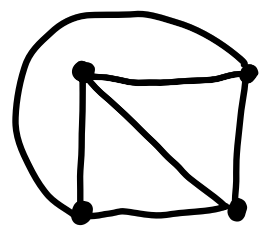
planar: can be drawn on a piece of paper w/out crossings

$\chi(K_1)=1$

$\chi(K_2)=2$

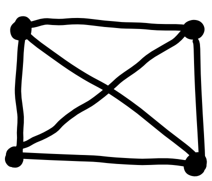


$\chi(K_3)=3$

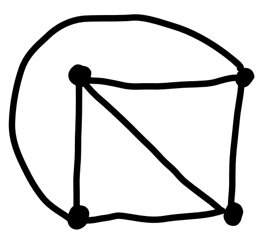


$\chi(K_4)=4$

Def 6.1.4: A graph G is planar if it has a drawing w/out crossings, called a <u>planar embedding</u> or a <u>plane graph</u>



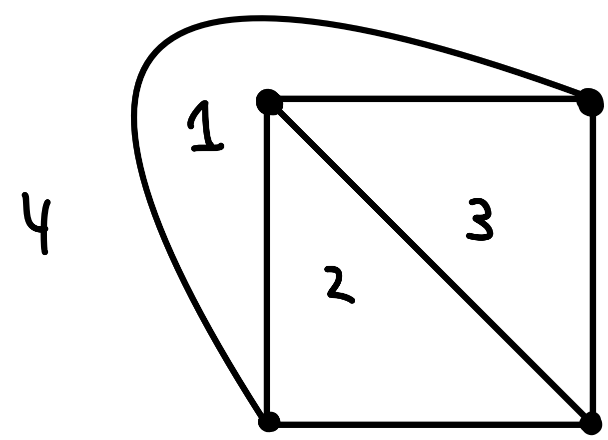
not a planar embedding



planar embedding

[so K_4 is planar]

Def: The faces of planar embedding are the maximal conn. regions of the plane not intersecting vertices and edges



Remark: It is surprisingly difficult to make some of these ideas rigorous. Need topology and the "Jordan Curve Theorem"

Prop 6.1.2: K_5 and $K_{3,3}$ are <u>not</u> planar