Announcements:

- Midterm 2 Wed.

  Wed. 10/18 7:00pm - 8:30pm in 217 Noyes Lab.

  See email for policies (covering Ch. 1-3 + circuit)

- Tues. problem session will be study session

- Wed. class will be review

Chapter 4: Connectivity & Paths

Idea of connectivity:

How many vertices/edges do we need to delete to form a disconnected graph?

Def 4.1.1: Let G be a graph

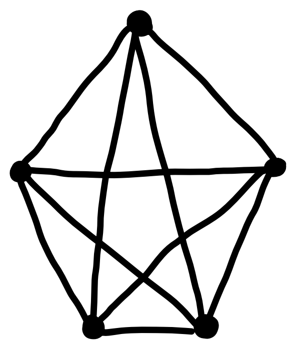
a) A vertex cut is a set $S \subseteq V(G)$ s.t $G \setminus S$ is disconn.

b) The (vertex) connectivity $\kappa(G)$ ("kappa") is the min. size of a vertex cut (OR $n-1$ if $\nexists$ vertex cut)

c) G is <u>k-connected</u> if $\kappa(G) \geq k$

Class activity: Find $\kappa(G)$ for the following graphs

a)



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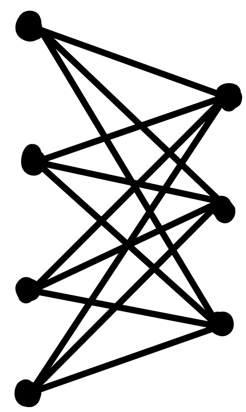
b)



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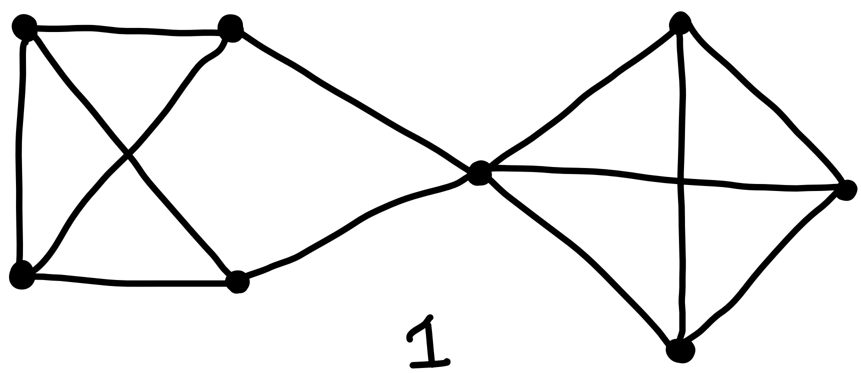
c)



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Def 4.1.7:

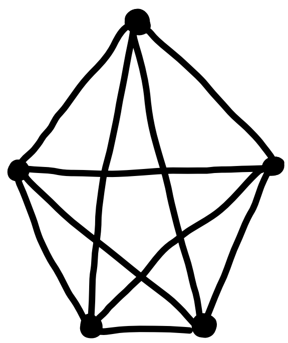
a') A <u>disconnecting set</u> is a set $F \subseteq E(G)$ s.t. $G \setminus F$ is disconn.

b') The <u>edge connectivity</u> $\kappa'(G)$ ← ' means "edges" is the min. size of a disconn. set (or $|E(G)|$ if $\nexists$ disconn. set)

c') G is <u>k-edge-connected</u> if $\kappa'(G) \geq k$

Class activity: Find $\kappa'(G)$ for the following graphs

a)



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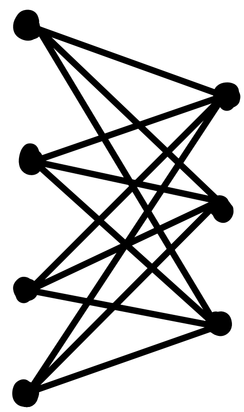
b)



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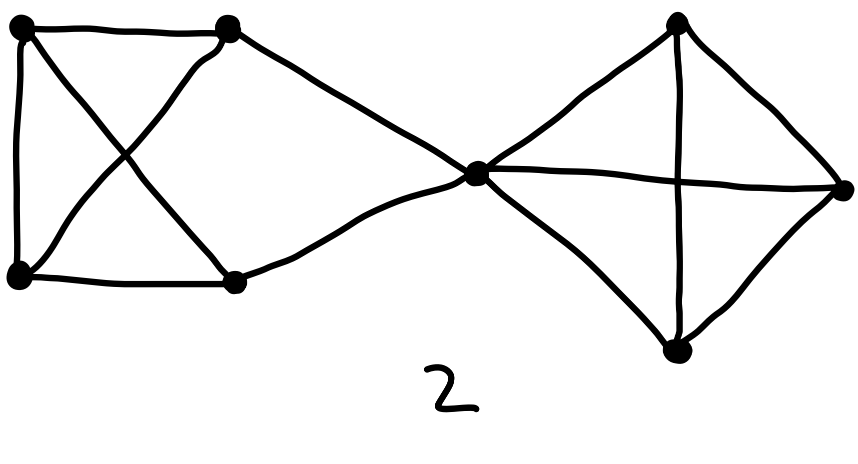
c)



2

d') An <u>edge cut</u> is a disconn. set F s.t $\exists S \subseteq V(G)$ where each edge in F has exactly one endpoint in S.
(every min'l disconn. set is an edge cut)

e') Every edge cut has the form

$[S, T] := \{ e \in E(G) \mid e$ has one endpoint in S and the other in $T \}$

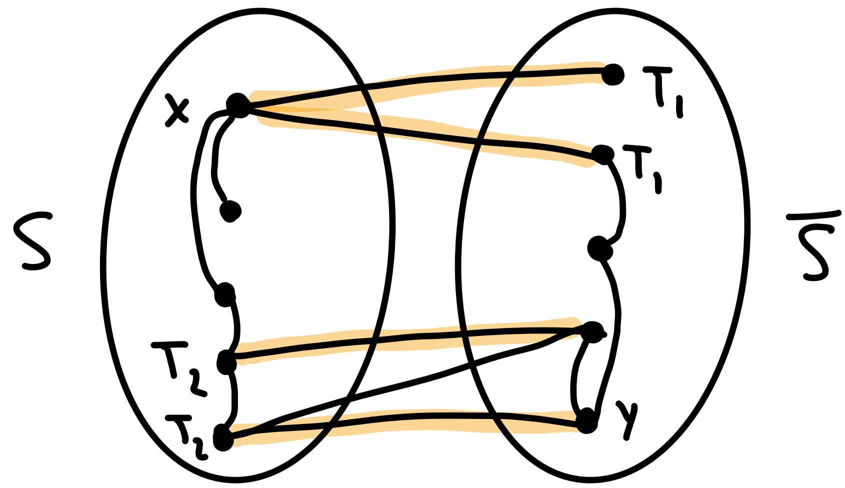
$\boxed{\overline{S} := V(G) \setminus S}$

Thm 4.1.9: Let G be a simple graph. Then,

$$\kappa(G) \le \kappa'(G) \le \delta(G)$$ min. deg.

Pf: If $v \in V(G)$ w/ $d(v) = \delta(G)$, then edges incident to v form an edge cut, so $\kappa'(G) \le \delta(G)$.

Now let $[S, \overline{S}]$ be a min'l edge cut. If $\exists$ nonadj. vertices $x \in S$, $y \in \overline{S}$, Let $T = \underbrace{N_{\overline{S}}(x)}_{T_1} \cup \underbrace{N_{S \setminus \{x\}}(\overline{S})}_{T_2}$



Every x,y-path passes from S to $\overline{S}$, and must pass thru. T as it does so, so T is a vertex cut.

Now, every vertex in T_1 has ≥ 1 edge to $x \in S$ while every vertex in T_2 has ≥ 1 edge to $\overline{S}$, so

$$\kappa(G) \leq |T| \leq |[S, \overline{S}]| = \kappa'(G)$$

On the other hand if G contains the complete $S, \overline{S}$-bigraph,

$$\kappa'(G) = |[S, \overline{S}]| = |S||\overline{S}| \geq n-1 \geq \kappa(G)$$

$\square$

Thm 4.1.11: If G is 3-regular, then $\kappa(G) = \kappa'(G)$

Pf: Let S be a minimum vertex cut.
Let H_1, H_2 be components in $G \setminus S$.
Since S is minimum, each $v \in S$ has a neighbor in H_1 and a neighbor in H_2, but since $d(v) = 3$, it can't have ≥ 2 neighbors in both. Thus, if v has only 1 edge to H_1, delete that edge; otherwise delete the edge to H_2. The result is an edge-cut of size $|S|$, so we're done $\square$