Announcements

Please join Gradescope course if you haven't already (code: 57YPR7)

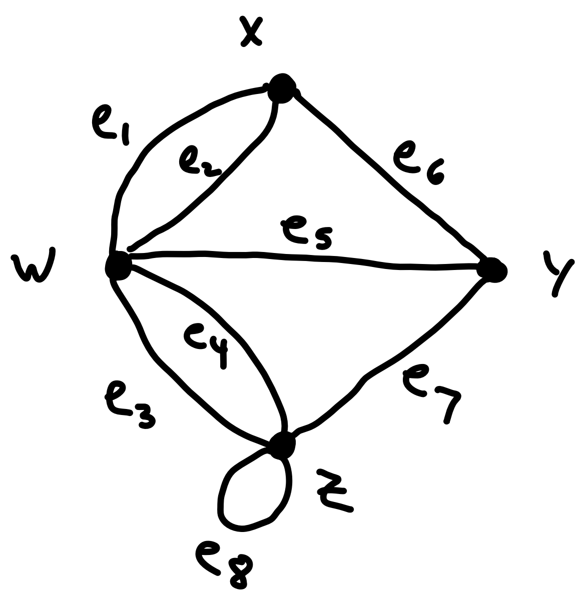
Midterms etc. mostly scheduled (see course website)

Today: Starting from scratch

Def 1.1.2: A graph G is a triple consisting of a nonempty vertex set $V(G)$, an edge set $E(G)$, and a relation that associates with each edge two vertices, called its <u>endpoints</u>

Ex 1.1.3:

G:



$V(G) = \{w, x, y, z\}$

$E(G) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$

e_1 has endpoints w & x

e_8 has endpoints z & z

Def 1.1.4:

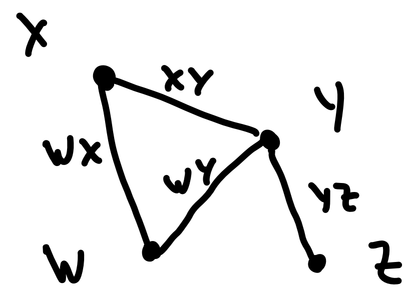
a) A <u>loop</u> is an edge whose endpoints are equal

Ex: e_8 is a loop

b) <u>Multiple edges</u> are edges w/ same pair of endpoints

Ex: e_1 & e_2 are mult. edges; e_3 & e_4 are mult. edges

c) A simple graph is a graph w/out loops or mult. edges
In this case, we often write uv (or vu) for the edge w/ end points u & v



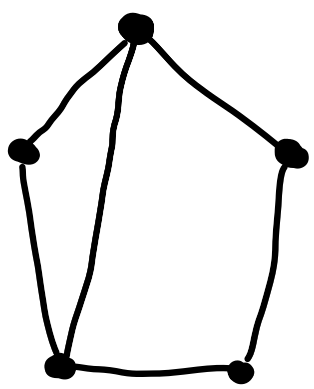
d) If two vertices u and v are the endpoints of an edge, we call them <u>adjacent</u> or <u>neighbors</u>

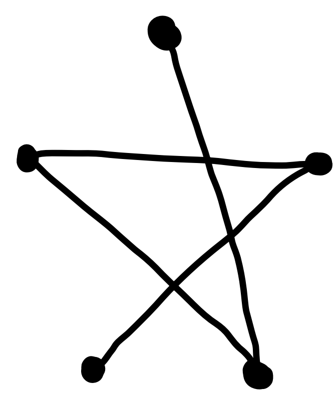
x & w are adjacent
y & z " "
w & z are not adjacent

Def 1.1.8:

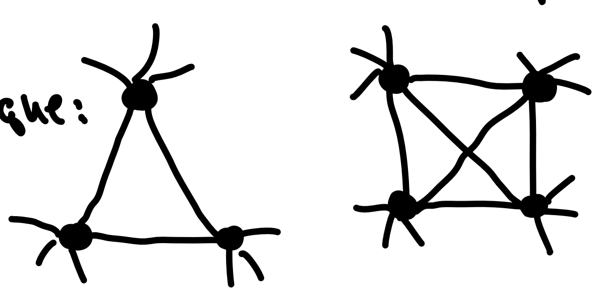
a) The complement $\overline{G}$ of a simple graph G is the simple graph with the same vertex set and edge set defined by

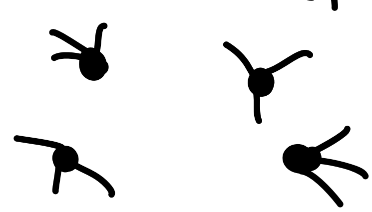
$$uv \in E(\overline{G}) \iff uv \notin E(G)$$

G : 

$\overline{G}$: 

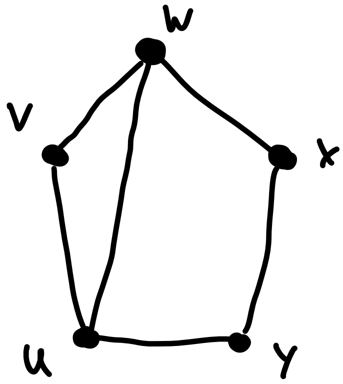
b) A <u>clique</u> is a set of pairwise adjacent vertices. An <u>independent</u> set is a set of pairwise nonadjacent vertices

Clique: 

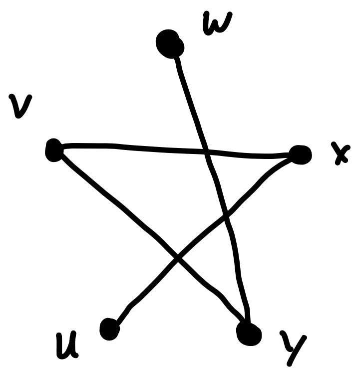
Independent set: 

Class activity: Find the largest clique and largest independent set in G and $\overline{G}$.

G :



$\overline{G}$:



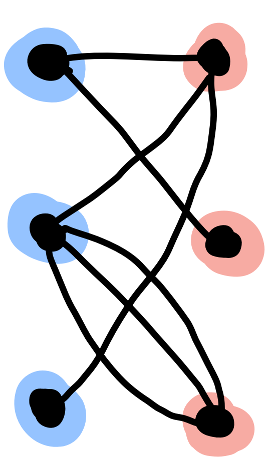
Clique: $\{v, w, u\}$

Indep. set $\{v, x\}, \{v, y\}, \ldots$

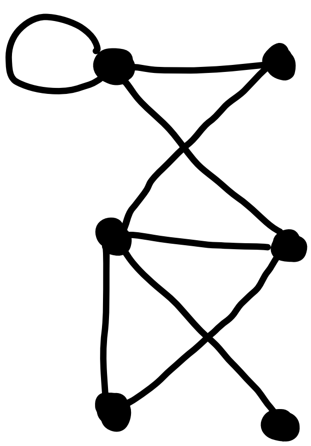
Clique: $\{v, x\}, \ldots$

Indep. set: $\{v, w, u\}$

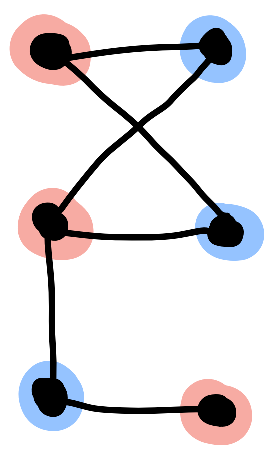
Def 1.10: A graph G is <u>bipartite</u> if $V(G)$ is the union of two independent sets



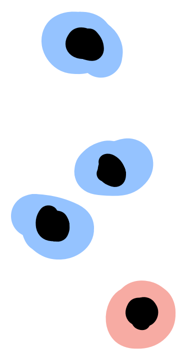
bipartite



not bipartite



bipartite



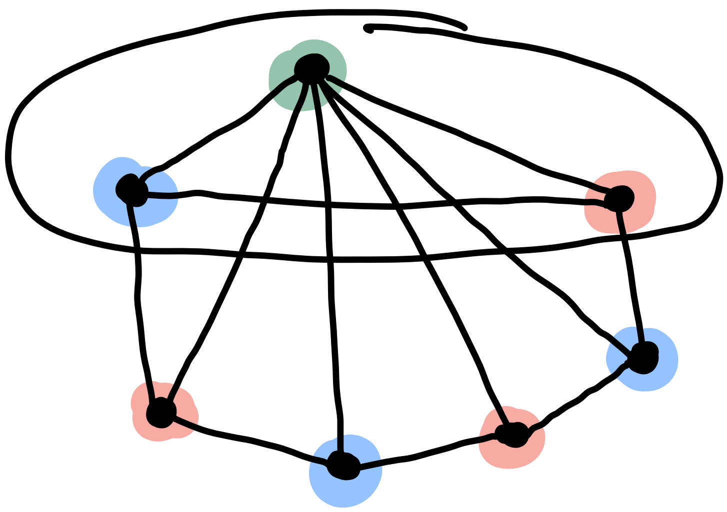
bipartite

Let's generalize this idea:

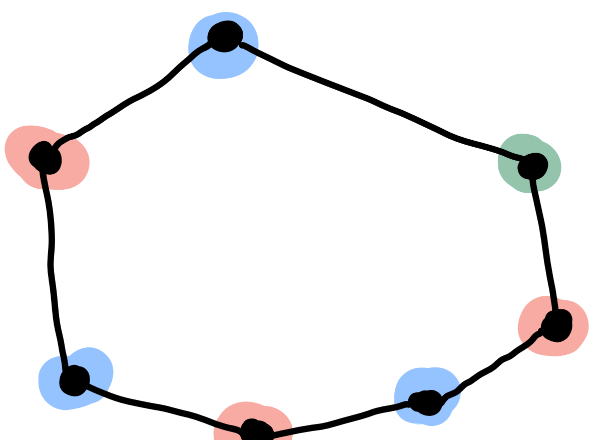
Def: 1.1.12:

a) G is <u>k-partite</u> if $V(G)$ can be expressed as the union of (at most) k independent sets

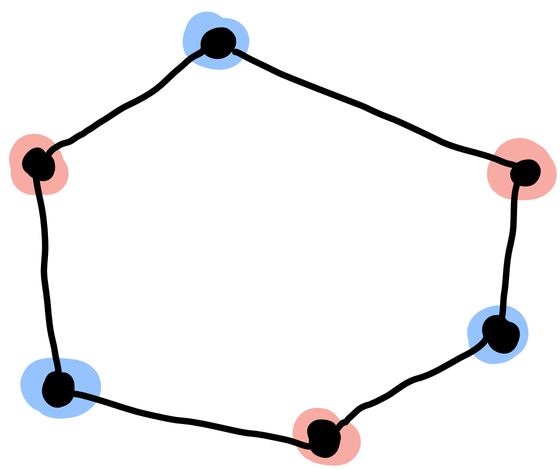
b) The chromatic number, "chi" $\chi(G)$, of G is the minimal value of k s.t. G is k-partite



$\chi(G) = 3$



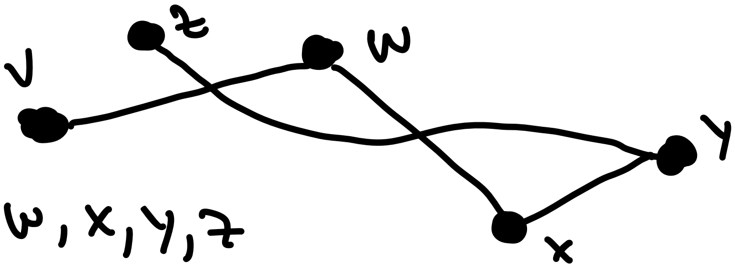
$\chi(G) = 3$



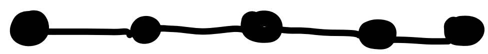
$\chi(G) = 2$

Def 1.1.15: $\boxed{|\text{largest clique}| \leq \chi(G) \leq |V(G)|}$

a) A <u>path</u> is a simple graph whose vertices can be ordered so that two vertices are adjacent if and only if they are consecutive in the list

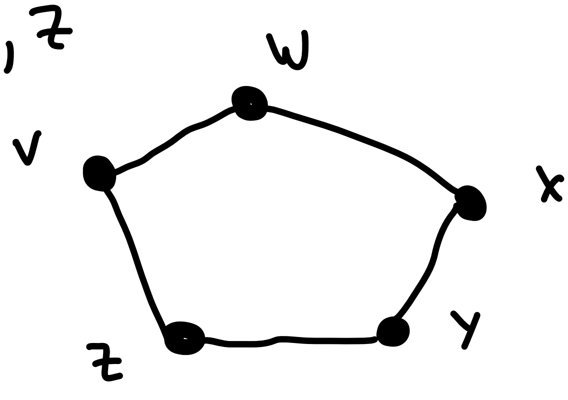


v, w, x, y, z



b) A <u>cycle</u> is a simple graph whose vertices can be ordered so that two vertices are adjacent if and only if they are consecutive in the list or if one is first and the other is last

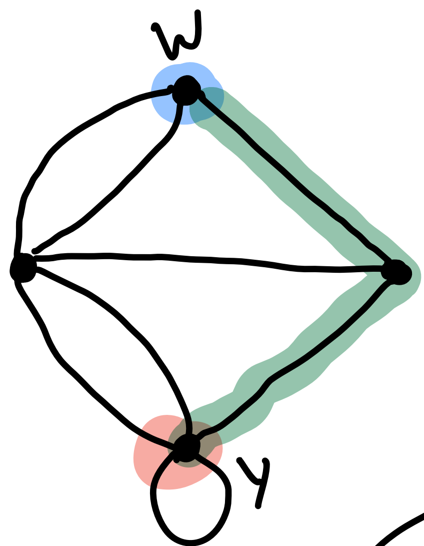
v, w, x, y, z



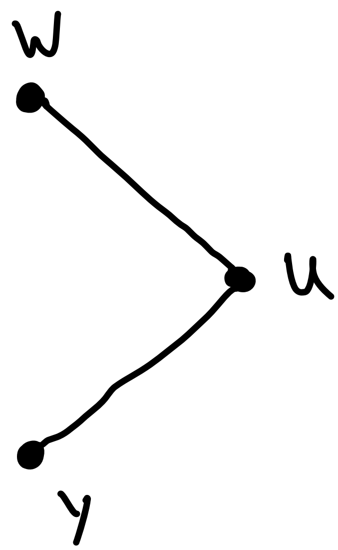
Def 1.1.16: a) A subgraph H of G is a graph where $V(H) \subseteq V(G)$, $E(H) \subseteq E(G)$, and the endpoints of each edge are the same. We write $H \subseteq G$.

b) G is connected if for every $v, w \in V(G)$, there exists a subgraph $H \subseteq G$ such that H is a path and $v, w \in V(H)$

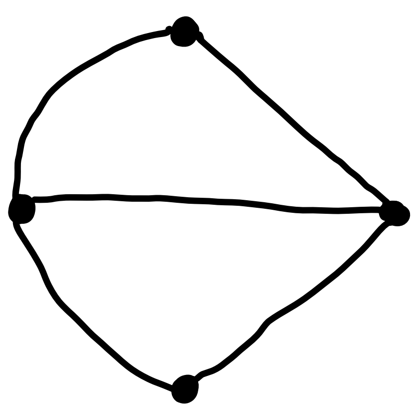
G:



H_1:



H_2



$H_1 \subseteq G$

$H_2 \subseteq G$

Adjacency Matrix

Let G be a loopless graph

Write $V(G) = \{v_1, \ldots, v_n\}$

$E(G) = \{e_1, \ldots, e_m\}$

Def 1.1.17

a) $v \in V(G)$ and $e \in E(G)$ are incident if v is an endpoint of e

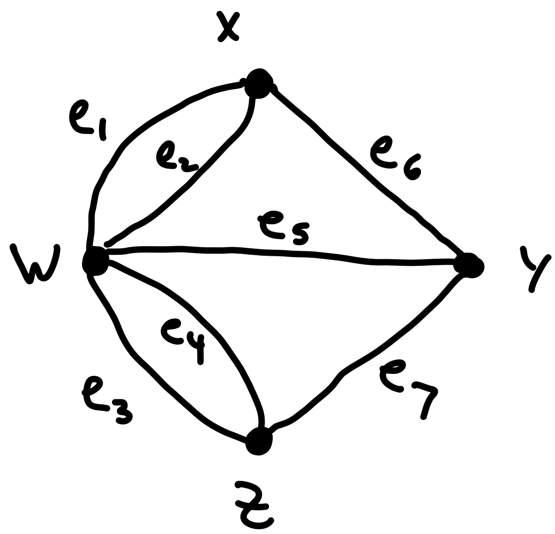
b) The adjacency matrix $A(G)$ is the $n \times n$ matrix where

a_{ij} = number of edges w/ endpoints v_i and v_j

c) The incidence matrix $M(G)$ is the $n \times m$ matrix where

$$m_{ij} = \begin{cases} 1 & \text{if } v_i \text{ is an endpoint of } e_j \\ 0 & \text{otherwise} \end{cases}$$

G :



$A(G) = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$

$M(G) = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$