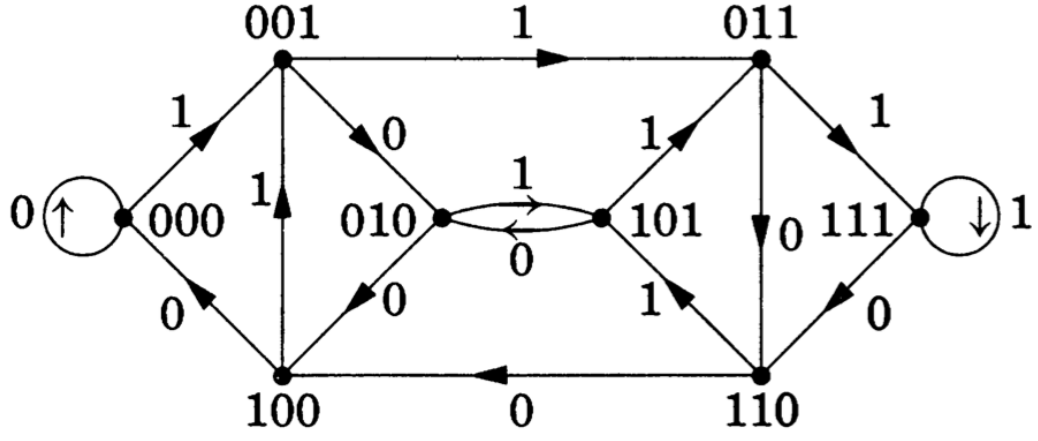
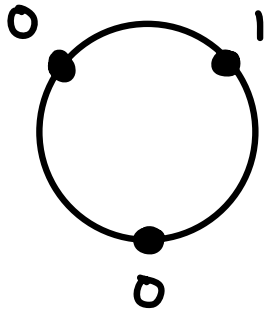
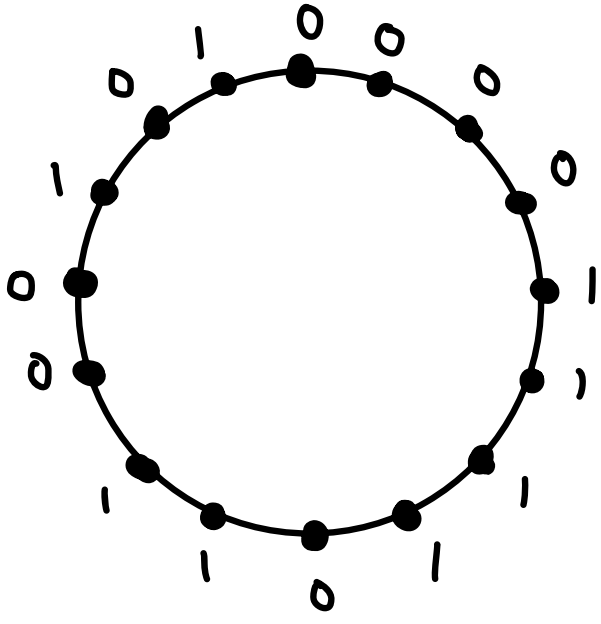
Announcements:

- Quiz today!
- Midterm 1: Wed. 9/20 7:00-8:30pm (Noyes 217)
  - Reference sheet allowed (two-sided)
  - See Monday's email for full policies

Recall: de Bruijn digraph D_n







Last time:

Eulerian circuit in D_n $\longleftrightarrow$ Cyclic arrangement w/ distinct n-strings

Thm 1.4.26: D_n has an Eulerian circuit

Pf: Every vertex of D_n has out-degree 2 (labelled 0 & 1) and in-degre 2 (both labelled by last digit of string)

To get to vertex $a_1 a_2 \cdots a_{n-1}$ from <u>any</u> vertex, follow edges labelled $a_1, a_2, \ldots, a_{n-1}$, so D_n is (strongly) connected. Thus, by Thm 1.4.24, D_n has an Eulerian circuit.

Def 1.4.27:

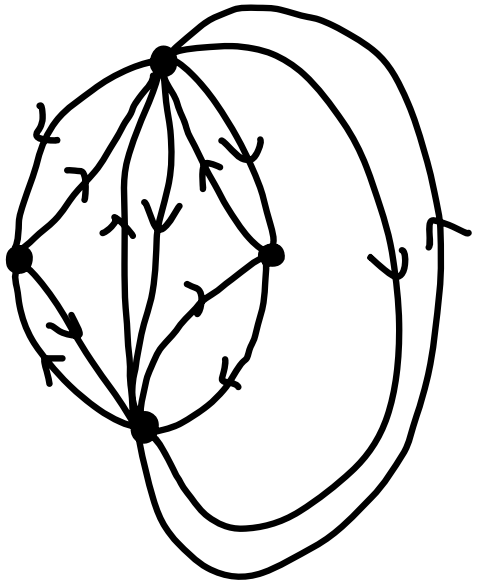
a) A digraph D is an <u>orientation</u> of a graph G if G is the underlying graph of D.

b) An <u>oriented graph</u> is an orientation of a simple graph

c) A <u>tournament</u> is an orientation of a complete graph

Class activity: Oriented graph? Tournament? Neither!

a)



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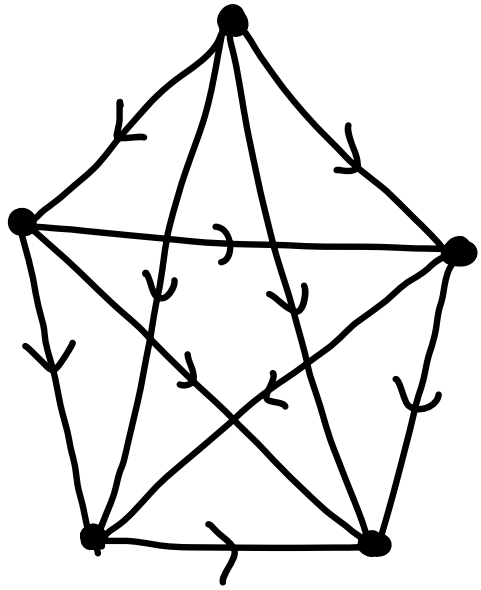
b)



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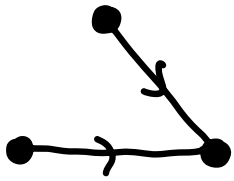
c)



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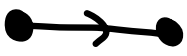
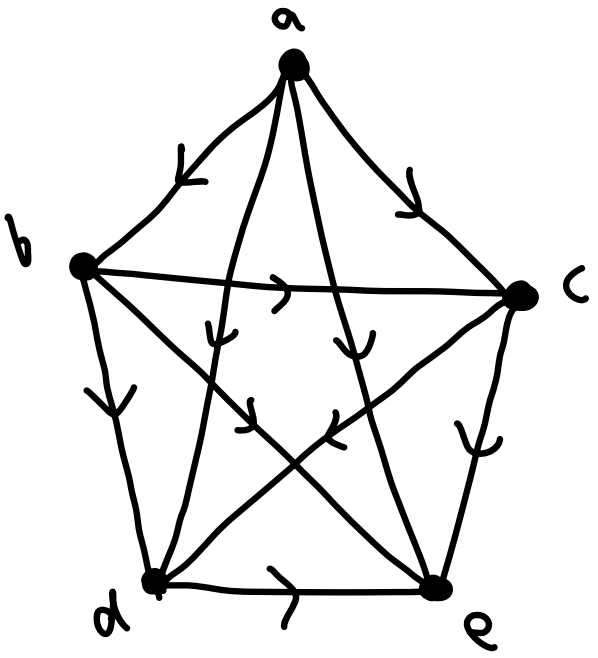
Reason for name "tournament":

Every player plays every other player ("round robin")

If a beats b, orient the edge like this



a b



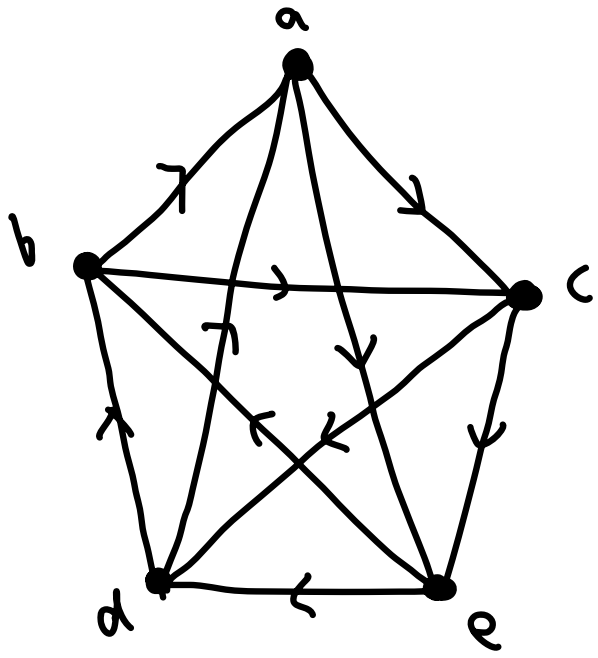
a beats b, c, d, e

b beats c, d, e

c beats d, e

d beats e

a is the champion & king



a beats c, e

b beats a, c

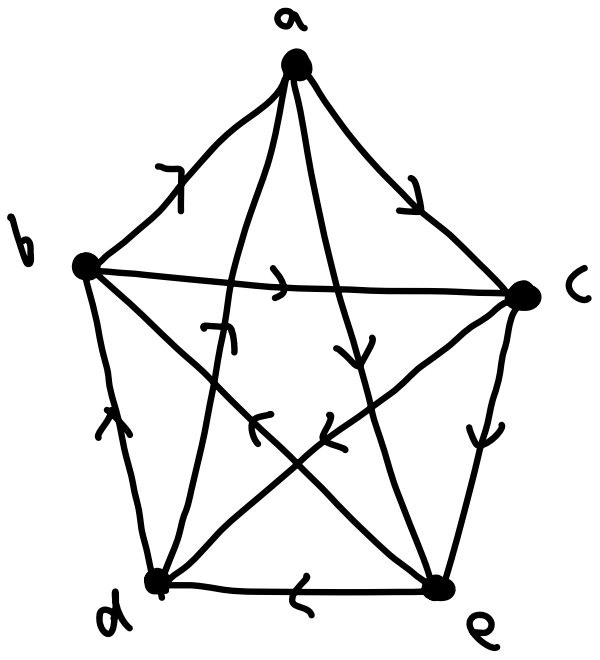
c beats d, e

d beats a, b

e beats b, d

Def 1.4.29: $v \in V(D)$ is called a <u>king</u> if there is a path of length ≤ 2 from v to every other vertex.

"a beats b who beats c"



a beats c

a beats e

a beats e beats b

a beats c beats d

So a is a king

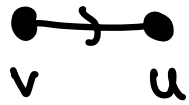
Prop 1.4.30: Every tournament T has at least one king

Pf: This follows from the following claim:

Claim: If v has maximum outdegree in T, then v is a king.

Pf of claim: Let $v \in V(T)$. If v is not a king, let $w \in V(T)$ s.t. there is no path from v to w of length ≤ 2.

Therefore: (i) 

(ii) If 

, then 

otherwise 

Hence, $d^+(w) > d^+(v)$. □