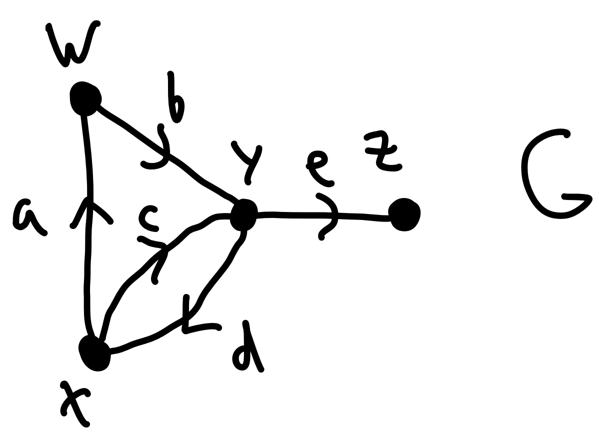
Announcements:

- H/w 2 graded; H/w 4 due 9/27 (2 weeks from today)
- Quiz 1 this Friday in class (20 mins)
  - Content: anything covered thru. today
- Midterm 1: Wed. 9/20 7:00-8:30pm (Noyes Lab. 217)
  - Reference sheet allowed (two-sided) otherwise, no resources allowed
  - See Monday's email for full policies

Class activity:



$$
\begin{array}{c|cccc}
 & w & x & y & z \\
\hline
w & 0 & 0 & 1 & 0 \\
x & 1 & 0 & 1 & 0 \\
y & 0 & 1 & 0 & 1 \\
z & 0 & 0 & 0 & 0
\end{array}
\qquad
\begin{array}{c|ccccc}
 & a & b & c & d & e \\
\hline
w & -1 & 1 & & & \\
x & 1 & & 1 & -1 & \\
y & & -1 & -1 & 1 & 1 \\
z & & & & & -1
\end{array}
$$

$A(G)$ $\qquad$ $M(G)$

g) For a vertex v,

$d^+(v)$: outdegree, # edges w/ tail v

$d^-(v)$: indegree, # edges w/ head v

$\delta^\pm(G)$: min out/indegree, $\Delta^\pm(G)$: max out/indegree

Successor: a vertex w s.t. $\exists$ an edge $v \to w$

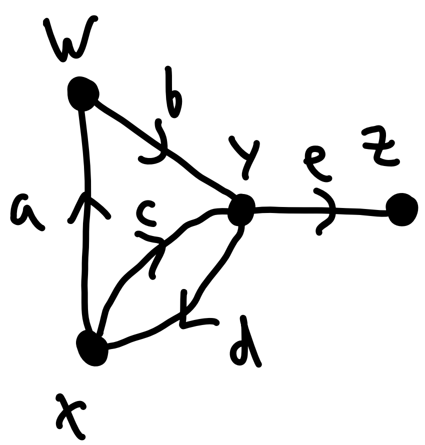
Predecessor: a vertex u s.t. $\exists$ an edge $u \to v$

$N^+(v)$: Out-nbhd/sucessor set, set of successors of v

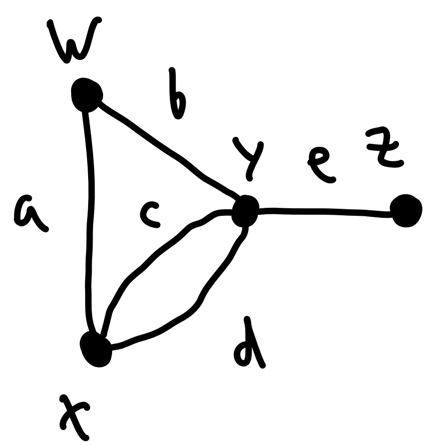
$N(v)$: In-nbhd/predecessor set, set of predecessors of v

Degree-sum formula: $e(G) = \sum_{v \in V(G)} d^+(v) = \sum_{v \in V(G)} d^-(v)$

h) The underlying graph of a digraph D is the graph G obtained by removing directions

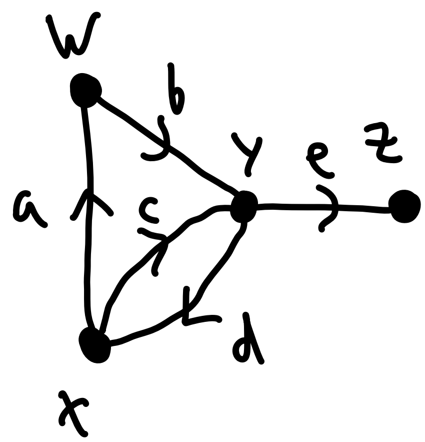


D



G

i) A digraph is weakly connected if the underlying graph is connected, and strongly connected if $\exists$ path from u to v $\forall$ vertices u,v



not strongly conn.

weakly conn.

Thm 1.4.24: D : digraph

D has an Eulerian circuit $\iff$ a) $d^+(v) = d^-(v) \ \forall v \in V(D)$

b) the underlying graph has ≤ 1 nontrivial component

D has an Eulerian trail $\iff$ a) $\sum_{v \in V(D)} |d^+(v) - d^-(v)| \leq 2$

b) the underlying graph has ≤ 1 nontrivial component

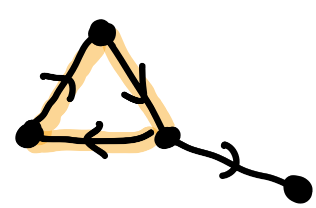
Pf (of first part):

$\Rightarrow$): If D has an Eulerian circuit W, then W must enter and leave each vertex the same number of times, and every edge of D must be in the same weakly connected. component.

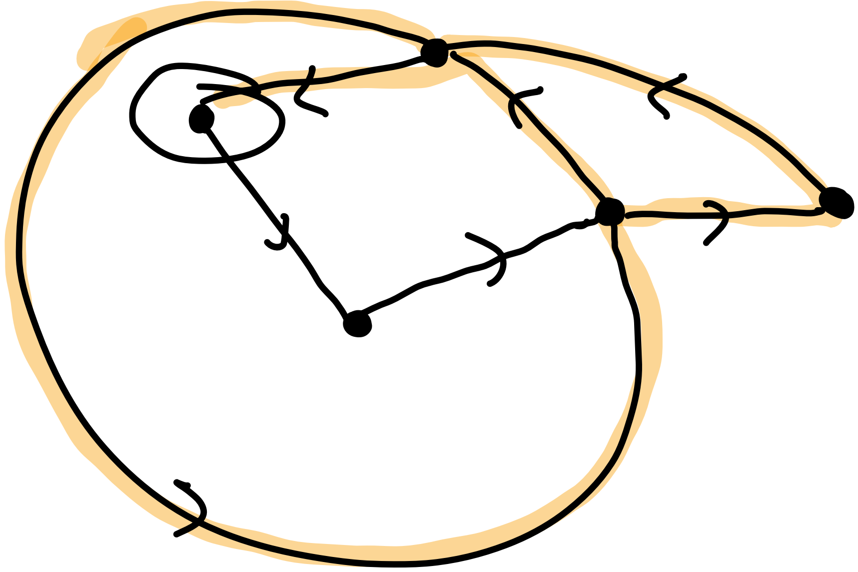
$\Leftarrow$): Let T be a maximal trail of D;

T must be closed i.e. a circuit since otherwise it enters its last vertex one more time than it leaves. Condition a) mean we can add another edge to T.

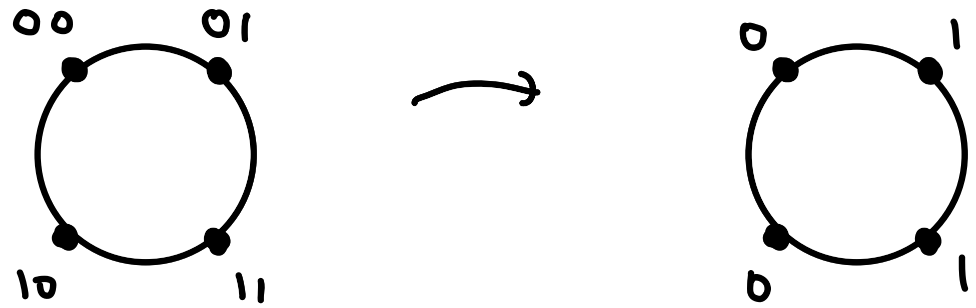
Suppose T doesn't use every edge of D; since $D \leq 1$ nontriv. conn. component, there must be some edge e of D not in T which has an endpoint v in T. Let T' be a cyclic ordering of T that starts and ends at v. Then adding e to the start or end of T' creates a longer trail. Contradiction. $\square$



Remark: If D has an Eulerian circuit, it's nontrivial component is <u>strongly</u> connected.



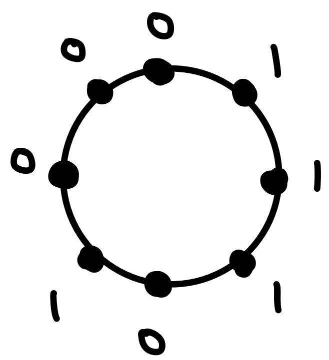
Application 1.4.25: de Bruijn cycles



Is there a cyclic arrangement of 2^n binary digits s.t. the 2^n strings of n consecutive digits are distinct?

$n=2$. Yes

$n=3$: Also yes



Let D_n be a digraph w/

$V(D_n)$ = binary strings of length $n-1$

$a \xrightarrow{x} b$ if $a = a_1 a_2 \cdots a_{n-1}$ i.e. the last $n-2$
$b = a_2 \cdots a_{n-1} x$ entries of a are the first $n-2$ entries of b

$n = 4$: