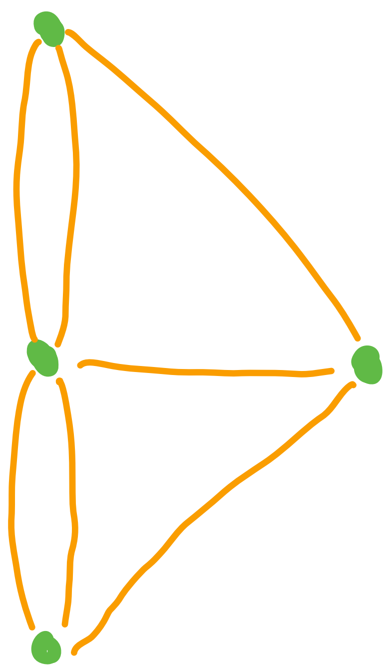
Math 412 — Graph Theory

Instructor: Andrew Hardt
(Andy)

Today: $\underbrace{\text{Syllabus}}$ + course overview
read this carefully;
you are responsible for
following course policies

Announcements:
- Sign up for course on Gradescope (see my email)
- First homework will be posted this Wednesday
  (due Wed. 8/30 @ 9am via Gradescope)



GRAPH THEORY
MATH 412 - Fall 2023

Lecture: *MWF 10:00am-10:50am; Henry Administrative Building 156*
Textbook: *Douglas B. West, Introduction to Graph Theory (2nd Edition), Chapters 1-7*
Course website: *https://andyhardt.github.io/412_F23/course_page.html*

Instructor: *Andrew Hardt (Office: CAB 69B; Email: ahardt@illinois.edu)*
Office Hours: *To be determined*
Problem Solving Sessions: Tuesdays, *Time and Location TBD*

The course website will be our primary resource for course information. Homework assignments and lecture notes will be posted there, as will updates to the syllabus and other information. You are responsible for all information and announcements posted there, as well as any made in class.

Prerequisites:
MATH 347, MATH 348, CS 374, or equivalent experience. Since this is a proof-based course, please talk to me if you don't have proof-writing experience.

Grading:
Course grades will be calculated using the following weights. Letter grades will be no stricter than A-: 90%, B-: 80%, C-: 70%, D-: 60%, but may be more lenient. Letter grades are only calculated for the entire semester; there are no letter grades for individual assignments.

Assignment	**Percent of Total**
Homework	20%
Quizzes	5%
Midterm Exams	45%
Final Exam	30%
Total	100%

Homework:
Homework assignments will be weekly, due at **9am on Wednesdays via Gradescope**. If you have not received an invite to the Gradescope course, please let me know immediately.

- Each homework assignment will contain five problems. Undergraduate students and graduate students registered for three credits should choose four out of these five problems to complete. Graduate students registered for four credits should complete all five problems.
- Assignments must be neat and legible, and in correct mathematical style. Most problems require proofs, and your submissions are expected to be clear and well-justified.
- Typesetting your homework using LaTeX is recommended, and will gain you 2 bonus points per assignment.
- Each student's lowest homework grade will be dropped.
- **Lateness policy:** Each student may have up to two "misses", homework assignments submitted less than 24 hours after the deadline that are still eligible for full credit. Any homework submitted more than 24 hours past the deadline (even by one minute!) will receive no credit, as will any late assignment after the first two.

LaTeX Resources:
One free, online resource for writing documents in LaTeX is Overleaf. There is also a convenient tool called Detexify for identifying the commands for specific symbols. Overleaf has extensive documentation (with many good examples), but please feel free to contact me if you need further help. Although it is possible to draw diagrams in LaTeX, using a package called TikZ, you are also welcome to draw diagrams by hand and insert pictures.

Problem-Solving Sessions:
These sessions are a supplement to office hours and are intended to provide collaborative time to work in groups on homework assignments. While I will be present at these sessions, you should think of them primarily as a time to work with other students.

Quizzes:
There will be four in-class quizzes throughout the semester.
Quiz 1: *To be determined*
Quiz 2: *To be determined*
Quiz 3: *To be determined*
Quiz 4: *To be determined*

These quizzes will focus on definitions and core results. Each student's lowest quiz score will be dropped.

Exams:
There will be three evening midterm exams, plus a final exam.
Midterm 1: *To be determined*
Midterm 2: *To be determined*
Midterm 3: *To be determined*
Final Exam: Thursday 12/14, 1:30pm-4:30pm, Location TBD

Make-ups:
If you have a documented and legitimate reason for missing an exam (i.e., illness, a death in the family, athletic travel for UIUC, etc), please contact me as soon as possible to make arrangements. This will typically be dealt with by weighting later exams more heavily and dropping the missed exam, but the exact solution may depend on individual circumstances.

Academic Honesty:
Students are expected to follow the University of Illinois student conduct code. While collaboration on homework assignments is allowed (and encouraged!), you must write up your solutions independently. All exams should reflect your own original work, with no consultation of outside resources or collaboration.

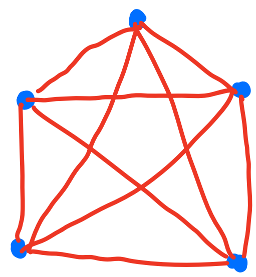
Disability Accommodations:
The University of Illinois at Urbana-Champaign is committed to providing equal access to educational opportunities via the Division of Disability Resources & Educational Services (DRES). If you feel that you may have a disability, you can contact DRES via e-mail at disability@illinois.edu or by phone at (217)-333-1970 to discuss possible accommodations. If you have existing accommodations through DRES, you should contact me as soon as possible to discuss how those accommodations will be implemented in this course.

Today: 3 examples of major graph theory results

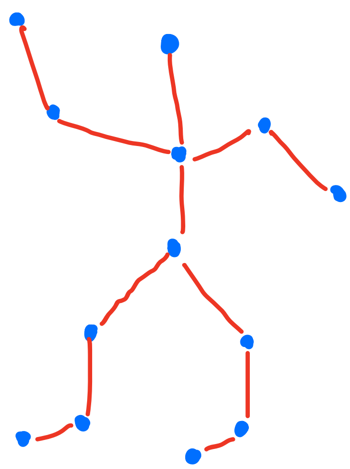
Graphs have vertices connected by edges

These are all graphs:

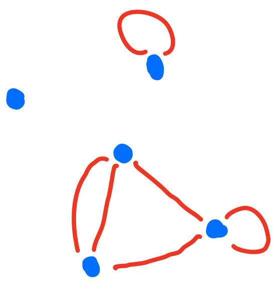
a)



b)

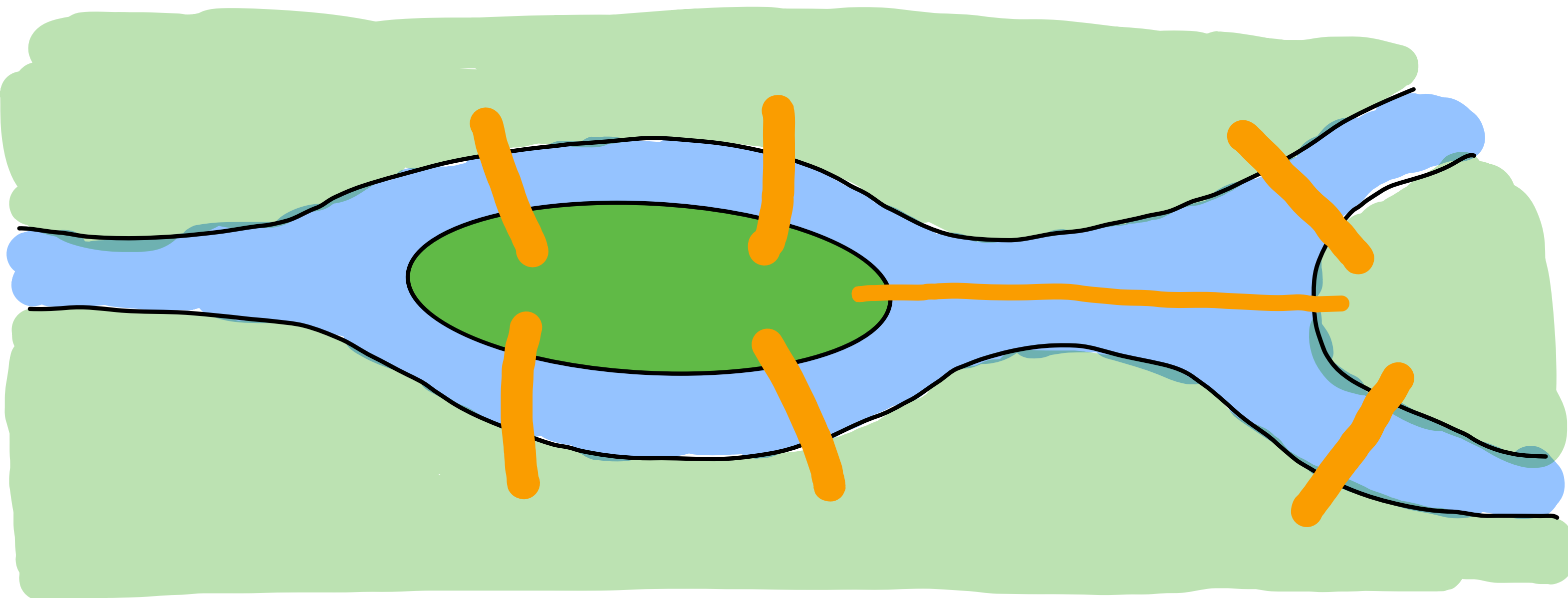


c)

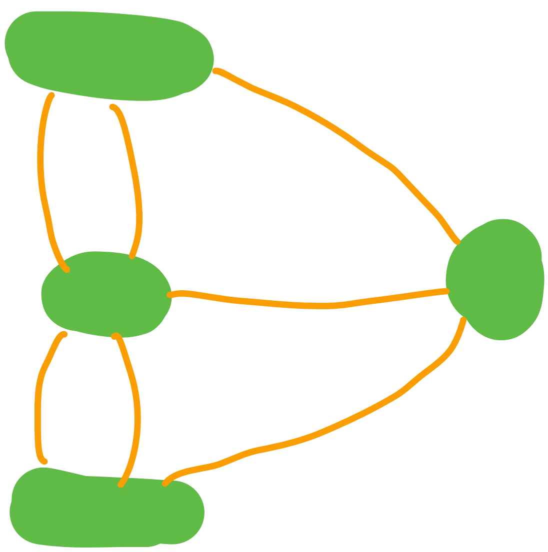


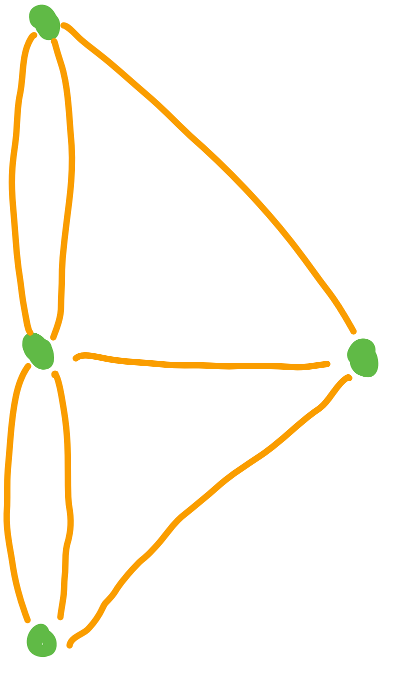
(We'll save formal definitions for next time)

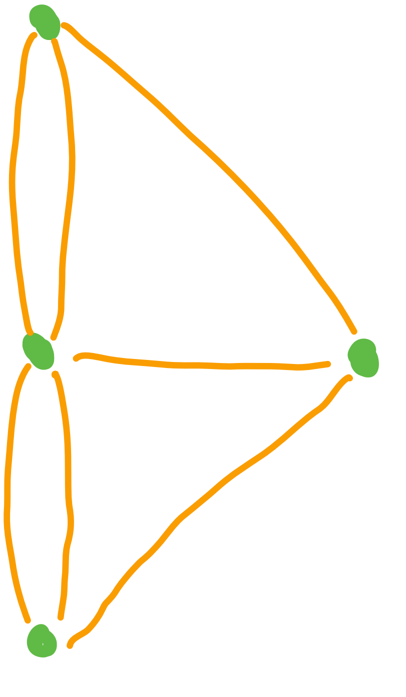
Example 1: Bridges of Königsberg



Question: Can we cross each bridge exactly once?







Theorem (Euler): It is possible to traverse every edge of a graph exactly once if and only if:

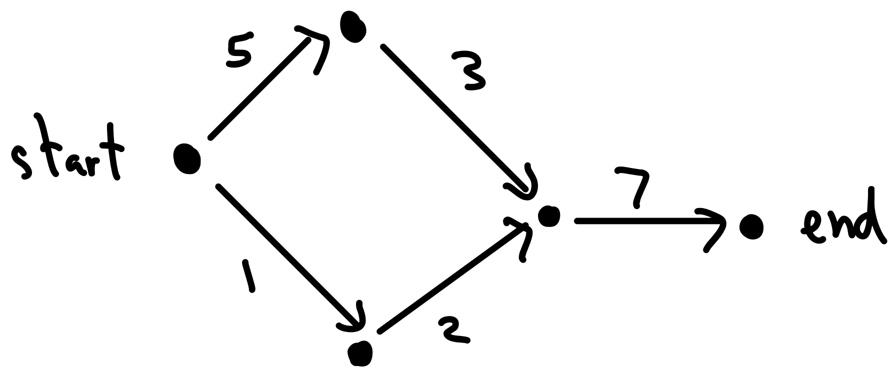
(i)

(ii)

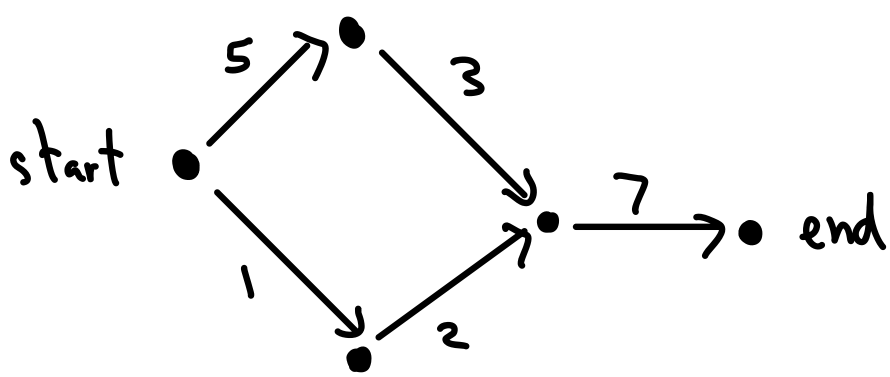
Example 2: Network flow

Consider a graph with

- a starting vertex (source)
- an ending vertex (sink)
- directions on all the edges
- water can flow along each edge up to a specified maximum (capacity)

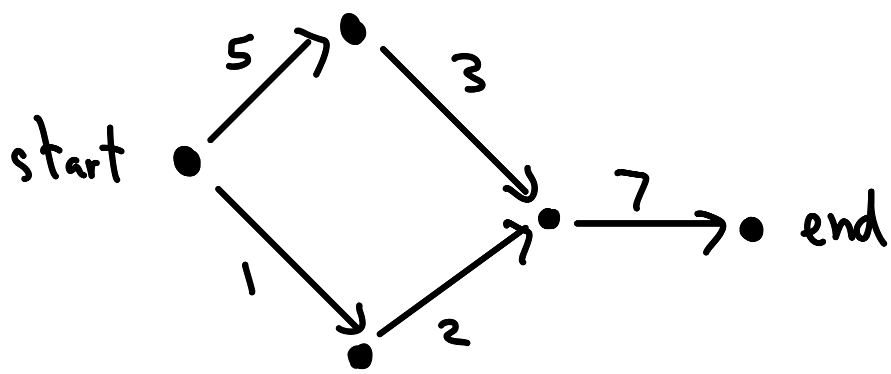


The maximum flow is the largest amount of water that can flow through this graph



Maximum flow:

The minimum cut is the smallest "bottleneck"



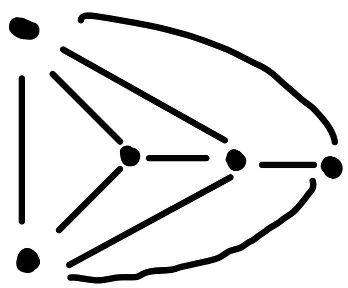
Minimum cut capacity:

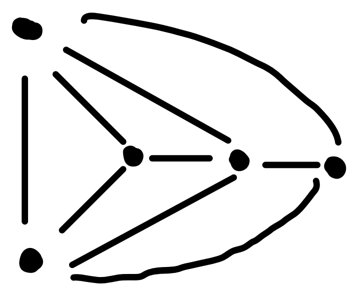
Theorem: The maximum flow always equals the capacity of the minimum cut

Proof: Ford-Fulkerson algorithm

Example 3: Four-Color Theorem

Color the vertices of a graph in such a way so that if two vertices share an edge, they are different colors

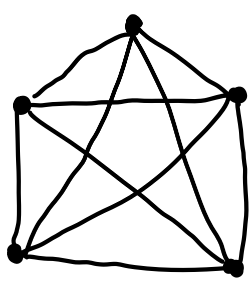




What is the minimum colors you need?

Ans: depends on the graph, but can be lots

e.g.

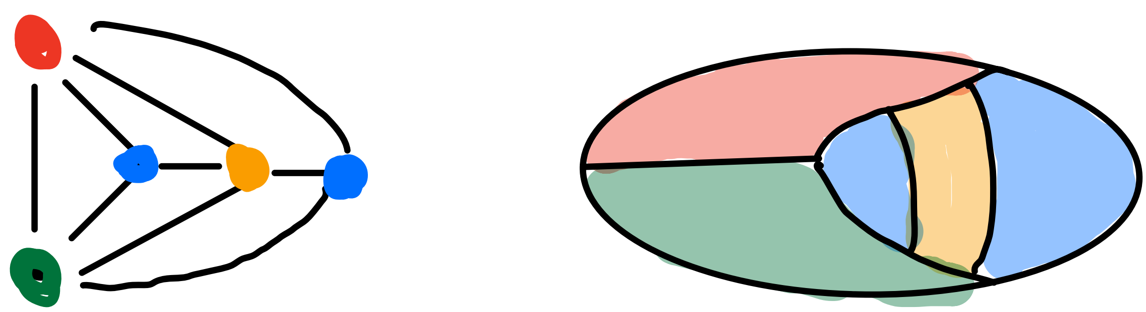


Needs 5 colors

Suppose the graph is <u>planar</u>

(i.e. the edges don't cross)

Then coloring a graph is like coloring a map



Theorem [Heawood, 1890]: Every planar graph can be colored with at most 5 colors

Theorem [Appel-Haken-Koch, 1977]: Every planar graph can be colored with at most 4 colors