Math 213, Spring 2026 **Practice Problems Final Exam**

Note: the distribution of these problems may not match the distribution of exam topics.

Problem §1.4: 15: Determine the truth value of each of these statements if the domain of each variable consists of all integers.

(a) $\forall n(n^2 \geq 0)$

(b) $\exists n(n^2 = 2)$

(c) $\forall n(n^2 \geq n)$

(d) $\exists n(n^2 < 0)$

Problem §2.3: 34: If f and $f \circ g$ are one-to-one, does it follow that g is one-to-one? Justify your answer.

Problem §2.2: 30: Can you conclude that $A = B$ if A, B, and C are sets such that

(a) $A \cup C = B \cup C$

(b) $A \cap C = B \cap C$

(c) $A \cup C = B \cup C$ and $A \cap C = B \cap C$

Problem §2.3: 34: If f and $f \circ g$ are one-to-one, does it follow that g is one-to-one? Justify your answer.

Problem §3.2: 18: Let k be a positive integer. Show that $f(k) = 1^k + 2^k + \cdots + n^k$ is $O(n^{k+1})$.

Problem §5.1: 12: Prove that

$$\sum_{j=0}^{n} \left(-\frac{1}{2}\right)^j = \frac{2^{n+1} + (-1)^n}{3 \cdot 2^n}$$

whenever n is a nonnegative integer.

Problem §5.1: 24: Prove that

$$\frac{1}{2n} \leq \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdots 2n}$$

whenever n is a positive integer.

Problem §5.1: 43: Prove that if $A_1, A_2, \ldots, A_n$ are subsets of a universal set U, then

$$\overline{\bigcup_{k=1}^{n} A_k} = \bigcap_{k=1}^{n} \overline{A_k}.$$

Problem §6.2 - 44: There are 51 houses on a street. Each house has an address between 1000 and 1099, inclusive. Show that at least two houses have addresses that are consecutive integers.

Problem §6.3 - 33: Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have the same number of men and women?

Problem §6.5 - 17: How many strings of 10 ternary digits (0, 1, or 2) are there that contain exactly two 0s, three 1s, and five 2s?

Problem §7.1 - 20: What is the probability that a five-card poker hand contains a royal flush, that is, the 10, jack, queen, king, and ace of one suit?

Problem §7.1 - 30: What is the probability that a player of a lottery wins the prize offered for correctly choosing five (but not six) numbers out of the six integers chosen at random from the integers between 1 and 40, inclusive?

Problem §7.2 - 38: A pair of dice is rolled in a remote location and when you ask an honest observer whether at least one die came up six, this honest observer answers in the affirmative.

(a) What is the probability that the sum of the numbers that came up on the two dice is seven, given the information provided by the honest observer?

(b) Suppose that the honest observer tells us that at least one die came up five. What is the probability the sum of the numbers that came up on the dice is seven, given this information?

Problem §8.1 - 28: Show that the Fibonacci numbers satisfy the recurrence relation $f_n = 5f_{n-4}+3f_{n-5}$ for $n = 5, 6, 7, \ldots$, together with the initial conditions $f_0 = 0, f_1 = 1, f_2 = 1, f_3 = 2$, and $f_4 = 3$. Use this recurrence relation to show that f_{5n} is divisible by 5, for $n = 1, 2, 3, \ldots$.

Problem §8.5 - 18: How many terms are there in the formula for the number of elements in the union of 10 sets given by the principle of inclusion–exclusion?

Problem §9.1 - 55: Let R be a relation that is reflexive and transitive. Prove that $R^n = R$ for all positive integers n. (Here, R^n means $R \circ \cdots \circ R$, with n copies of R).

Problem §9.3 - 23-26: List the ordered pairs in the relations represented by the directed graphs. (see Rosen)

Problem §10.2.20: Draw the following graphs: $K_7, K_{1,8}, K_{4,4}, C_7, W_7, Q_4$

Problem §10.2.21-25: Determine whether the graph is bipartite

Problem §10.3.17: Draw an undirected graph represented by the given adjacency matrix (see Rosen)

Problem §10.4.2: Does each of these lists of vertices form a path in the following graph? (See Rosen!) Which paths are simple? Which are circuits? What are the lengths of those that are paths?

(a) a, b, e, c, b

(b) a, d, a, d, a

(c) a, d, b, e, a

(d) a, b, e, c, b, d, a

Problem §10.5.46: Show that the Petersen graph G (see Rosen!) does not have a Hamilton circuit, but that the subgraph obtained by deleting a vertex v, and all edges incident with v, does have a Hamilton circuit.

Problem §10.6.18: Is a shortest path between two vertices in a weighted graph unique if the weights of edges are distinct?

Problem §10.7.15: If a connected planar simple graph has e edges and v vertices with $v \geq 3$ and no circuits of length three, prove that $e \leq 2v - 4$.

Problem §11.1.2: Which of these graphs are trees?

Problem §11.1.14: Show that a simple graph is a tree if and only if it is connected but the deletion of any of its edges produces a graph that is not connected.

Problem §11.2.1: Build a binary search tree for the words banana, peach, apple, pear, coconut, mango, and papaya using alphabetical order.

Problem §11.2.33: Draw a game tree for nim if the starting position consists of two piles with two and three stones, respectively. When drawing the tree represent by the same vertex symmetric positions that result from the same move. Find the value of each vertex of the game tree. Who wins the game if both players follow an optimal strategy?

Problem §11.4.4-6: Find a spanning tree for the graph shown (see Rosen)

Problem §11.5.2,4,7,8: Use Prim's algorithm and Kruskal's algorithm to find minimum spanning trees for the given weighted graph. (see Rosen)