Note: the distribution of these problems may not match the distribution of exam topics.

Problem §6.2: 31: Show that there are at least six people in California (population: 37 million) with the same three initials who were born on the same day of the year (but not necessarily the same year). Assume that everyone has three initials.

Problem §6.2: 40: Prove that at a party where there are at least two people, there are two people who know the same number of other people there.

Problem §6.3 - 22(d,e,f): How many permutations of the letters $ABCDEFGH$ contain

(d) the strings AB, DE, and GH?

(e) the strings CAB and BED?

(f) the strings BCA and ABF?

Problem §6.4 - 20: Suppose that k and n are integers with $1 \leq k < n$. Prove the **hexagon identity**

$$\binom{n-1}{k-1}\binom{n}{k+1}\binom{n+1}{k} = \binom{n-1}{k}\binom{n}{k-1}\binom{n+1}{k+1}$$

which relates terms in Pascal's triangle that form a hexagon.

Problem §6.5 - 16(a,d): How many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 29,$$

where x_i, for $i = 1, 2, 3, 4, 5, 6$, is a non-negative integer such that

(a) $x_i > 1$ for $i = 1, 2, 3, 4, 5, 6$?

(d) $x_1 < 8$ and $x_2 > 8$?

Problem §6.5 - 30: How many different strings can be made from the letters in $MISSISSIPPI$, using all the letters?

Problem §7.1 - 18: What is the probability that a five-card poker hand contains a straight flush, that is, five cards of the same suit of consecutive kinds?

Problem §7.1 - 24(a): Find the probability of winning a lottery by selecting the correct six integers, where the order in which these integers are selected does not matter, from the positive integers not exceeding 30.

Problem §7.2 - 5: A pair of dice is loaded. The probability that a 4 appears on the first die is 2/7, and the probability that a 3 appears on the second die is 2/7. Other outcomes for each die appear with probability 1/7. What is the probability of 7 appearing as the sum of the numbers when the two dice are rolled?

Problem §7.3 - 11: An electronics company is planning to introduce a new camera phone. The company commissions a marketing report for each new product that predicts either the success or the failure of the product. Of new products introduced by the company, 60% have been successes. Furthermore, 70% of their successful products were predicted to be successes, while 40% of failed products were predicted to be successes. Find the probability that this new camera phone will be successful if its success has been predicted.

Problem §8.1 - 8:

(a) Find a recurrence relation for the number of bit strings of length n that contain three consecutive 0s.

(b) What are the initial conditions?

(c) How many bit strings of length seven contain three consecutive 0s?

Problem §8.2 - 26(a,c): What is the general form of the particular solution guaranteed to exist by Theorem 6 of the linear nonhomogeneous recurrence relation $a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3} + F(n)$ if

(a) $F(n) = n^2$?

(c) $F(n) = n2^n$?

Problem §8.5 - 24: Find the probability that when a fair coin is flipped five times tails comes up exactly three times, the first and last flips come up tails, or the second and fourth flips come up heads.

Problem §8.6 - 16: A group of n students is assigned seats for each of two classes in the same classroom. How many ways can these seats be assigned if no student is assigned the same seat for both classes?

Problem §9.1 - 42: List the 16 different relations on the set $A = \{0, 1\}$.

Problem §9.1 - 44(a,c,d,f): Which of the 16 relations on $A = \{0, 1\}$, which you listed in Exercise 42, are reflexive? Symmetric? Antisymmetric? Transitive?

Problem §9.3 - 6: How can the matrix representing a relation R on a set A be used to determine whether the relation is asymmetric?

Problem §9.3 - 13: Let R be the relation represented by the matrix

$$M_R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

Find the matrices representing $R^{-1}, \overline{R}, R \circ R$.