Note: the distribution of these problems may not match the distribution of exam topics.

Problem §1.3: 14: Determine whether $(\neg p \wedge (p \implies q)) \implies \neg q$ is a tautology.

Problem §1.4: 16: Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

(a) $\exists x(x^2 = 2)$

(b) $\exists x(x^2 = -1)$

(c) $\forall x(x^2 + 2 > 1)$

(d) $\forall x(x^2 \neq x)$

Problem §2.1: Let $A = \{2, 6\}$ and $B = \{3, 1\}$.

(a) Find $\mathcal{P}(A)$ and $|\mathcal{P}(A)|$.

(b) Find $A \times B$.

(c) Is $A \times B = B \times A$? Why or why not?

Problem §2.1: Let $C = \{n \in \mathbb{N} : n < 6\}$ and $D = \{1, 3, 5\}$.

(a) Write C in set roster notation.

(b) Is $D \subseteq C$? Why or why not?

(c) Draw a Venn diagram representing sets C and D. (*Hint: This diagram should reflect the relationship that you determined in part (b).*)

Problem §2.2: Prove that $A \cup (A \cap B) = A$ by showing that each set is a subset of the other.

Problem §2.3: Consider the function $f : \mathbb{Z} \to \mathbb{Z}_{\geq 0}$ defined by $z \mapsto |z| + 1$.

(a) Is f one-to-one? Why or why not?

(b) Is f onto? Why or why not?

Problem §3.1: 61: Write the deferred acceptance algorithm in pseudocode.

Problem §3.2: Give a big-O estimate for $f(x) = (7n^n + n2^n + 3^n)(n! + 3^n)$. For the function $g(x)$ in your estimate $O(g(x))$, use a simple function g of the smallest order.

Problem §3.2: Use the definition of "$f(x)$ is $O(g(x))$" to show that $f(x) = x^4 + 7x^3 + 8$ is $O(x^4)$.

Problem §5.1: 10:

(a) Find a formula for
$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n(n+1)}$$
by examining the values of this expression for small values of n.

(b) Prove the formula you conjectured in part (a).

Problem §5.1: 18: Let $P(n)$ be the statement that $n! < n^n$, where n is an integer greater than 1.

(a) What is the statement $P(2)$?

(b) Show that $P(2)$ is true, completing the basis step of the proof.

(c) What is the inductive hypothesis?

(d) What do you need to prove in the inductive step?

(e) Complete the inductive step.

(f) Explain why these steps show that this inequality is true whenever n is an integer greater than 1.

Problem §5.1: 22: For which nonnegative integers is $n^2 \le n!$? Prove your answer.

Problem §5.1: 36: Prove that 21 divides $4^{n+1} + 5^{2n-1}$ whenever n is a positive integer.

Problem §5.1: 38: Prove that if $A_1, A_2, \ldots, A_n$ and $B_1, B_2, \ldots, B_n$ are sets such that $A_j \subseteq B_j$ for $j = 1, 2, \ldots, n$, then
$$\bigcup_{j=1}^{n} A_j \subseteq \bigcup_{j=1}^{n} B_j.$$

Problem §5.1: 57: Use mathematical induction to prove that the derivative of $f(x) = x^n$ equals nx^{n-1} whenever n is a positive integer.

Problem §5.2: 31: Show that strong induction is a valid method of proof by showing that it follows from the well-ordering property.

Problem §6.1: 28: How many license plates can be made using either three digits followed by three uppercase English letters or three uppercase English letters followed by three digits?

Problem §6.1: 37: How many functions are there from the set $\{1, 2, ..., n\}$, where n is a positive integer, to the set $\{0, 1\}$

(a) That are one-to-one?

(b) That assign 0 to both 1 and n?

(c) That assign 1 to exactly one of the positive integers less than n?

Problem §6.2: 31: Show that there are at least six people in California (population: 37 million) with the same three initials who were born on the same day of the year (but not necessarily the same year). Assume that everyone has three initials.

Problem §6.2: 40: Prove that at a party where there are at least two people, there are two people who know the same number of other people there.