Announcements

HW2 posted (due Wed. @ 9am via Gradescope)

Quiz 1: score will be higher of $\underbrace{Q1+Q2+Q3}_{\text{each question worth 4 pts.}}$ or $\frac{3}{2}(Q2+Q3)$

Recall: A <u>function</u> $f: A \to B$ is an assignment of exactly one elt. of B to each elt. of A

- A is the <u>domain</u> of f
- B is the <u>codomain</u> of f
- The <u>range/image</u> of f is the set $\{f(a) \mid a \in A\}$
- If $a \in A$, $f(a)$ is the <u>image</u> of a under f
- If $b \in B$, the <u>preimage</u> of b under f is the set

$$f^{-1}(b) = \{a \in A \mid f(a) = b\}$$

Ex: $A = \{a, b, c\}$ $B = \{x, y, z\}$

$f: A \to B$

$f(a) = x$ $f(b) = z$ $f(c) = x$

f has:

- domain A
- codomain B
- range $\{x, z\}$

The image of c is x

The preimage of x is $\{a, c\}$

The preimage of y is $\emptyset$

Can also do image/preimage of sets

Def: Let $f: A \to B$. Let $C \subseteq A$ and $D \subseteq B$

The image of C is $f(C) = \{f(c) \mid c \in C\}$

The preimage of D is $f^{-1}(D) = \{a \in A \mid f(a) \in D\}$

Ex (cont): $f(\{a, c\}) = \{x\}$

$f^{-1}(\{x, z\}) = A$

Def: $f: A \to B$

f is <u>one-to-one</u>/<u>injective</u> if whenever $a \neq b, f(a) \neq f(b)$

f is <u>onto</u>/surjective if $f(A) = B$ range

f is <u>bijective</u> if it is injective and surjective

Ex (cont.):

f is not injective since $f(a) = x = f(c)$, but $a \neq c$

f is not surjective since $y \notin f(A)$

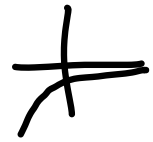
Ex: $g: \mathbb{R} \to \mathbb{R}$

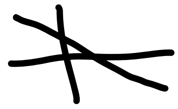
$g(x) = x + 1$

g is injective since if $g(x) = g(y)$ then $x+1 = y+1$, so $x = y$

g is surjective since if $z \in \mathbb{R}$, $g(z-1) = z$

Note: every function $f: \mathbb{R} \to \mathbb{R}$

that is strictly increasing 

or strictly decreasing 

is injective

Bijections have inverse functions

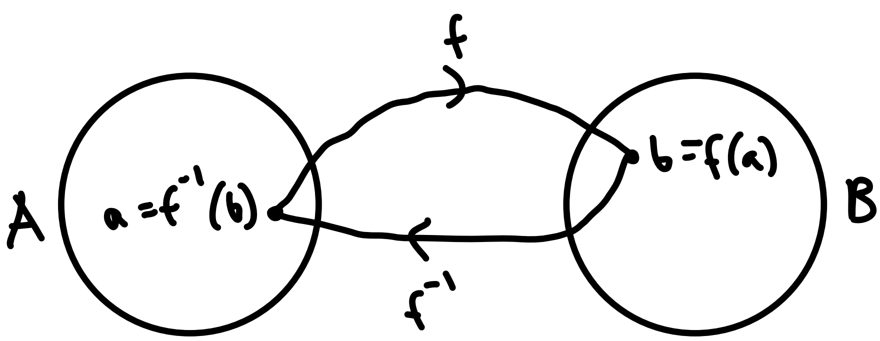
* See below for rigorous def'n

$f: A \to B$ bijection

$f^{-1}: B \to A$ (also a bijection)

f^{-1} "undoes" f: if $f(a) = b$, then $f^{-1}(b) = a$

We call a function with an inverse <u>invertible</u>



Ex:

a) $f: \mathbb{R}_+ \to \mathbb{R}_+$, $f(x) = x^2$ ← set of pos. real nums.

is invertible w/ $f^{-1}(x) = \sqrt{x}$ ← pos. sqrt.

b) $A = \{a, b, c\}$ $\quad f: A \to A$

$f(a) = b \quad f(b) = c \quad f(c) = a$

is invertible w/

$f^{-1}(a) = c \quad f^{-1}(b) = a \quad f^{-1}(c) = b$

Composition: apply functions in sequence

Let $f: A \to B \qquad g: B \to C$

need these to be the same

Then $g \circ f: A \to C$ is given by

$$g \circ f(a) = g(f(a))$$

Ex: $f: \mathbb{Z} \to \mathbb{Z} \qquad g: \mathbb{Z} \to \mathbb{N}$

$f(x) = x + 1 \qquad g(x) = x^2$

$g \circ f: \mathbb{Z} \to \mathbb{N}$

$(g \circ f)(x) = (x+1)^2$

$f \circ g$ is not defined since $\mathrm{dom}(f) \neq \mathrm{codom}(g)$

See textbook for more examples

* More on inverse functions

Let $f: A \to B$ and $g: B \to A$ be functions

f and g are inverse functions if

$\forall a \in A, \ g(f(a)) = a$ ("g undoes f")

and

$\forall b \in B, \ f(g(b)) = b$ ("f undoes g")

We need both of these to hold for f and g to be inverses

All bijections have inverses, and no non-bijections have inverses