Announcements

Quiz 1 today! (15 mins); front and back!

H/W 1 due this Friday @ 9am via Gradescope

Office hours:

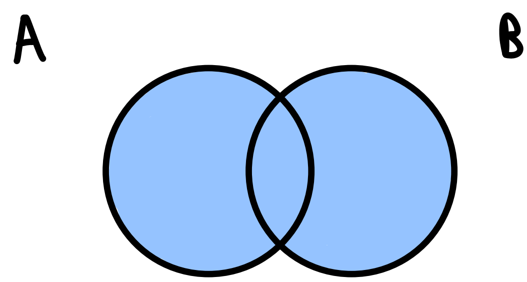
- Monday 2:00 pm – 3:20 pm (problem session), Everitt 2101
- Wednesday 2:00 pm – 2:50 pm
- Friday 11:00 am – 11:50 am

Harker 204C

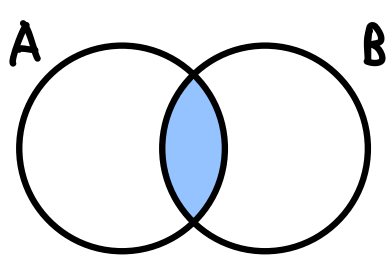
Working through §2.2: Set Operations

Venn diagrams and set identities:

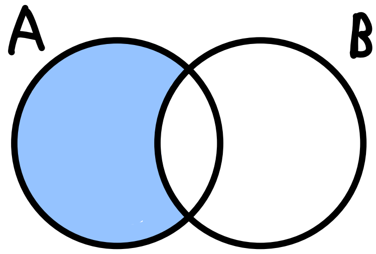
$A \cup B$



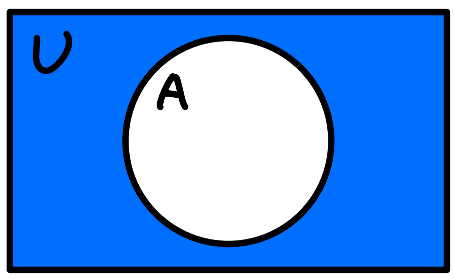
$A \cap B$



$A \setminus B$



$\bar{A}$



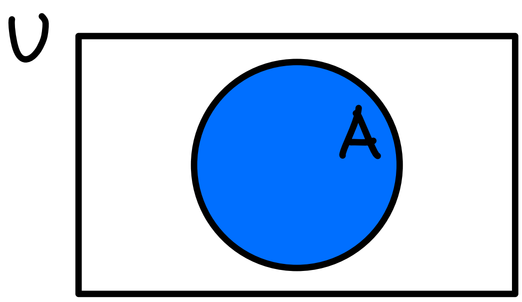
Set identities

Let A, B, C be sets, and let U be the universal set (always have $A, B, C \subseteq U$)

1) Identity laws

$A \cap U = A$

$A \cup \phi = A$



2) Domination laws

$A \cup U = U$

$A \cap \phi = \phi$

3) Idempotent laws

$A \cup A = A$

$A \cap A = A$

4) Complementation law

$$\overline{\overline{A}} = A$$

5) Commutative laws

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

6) Associative laws

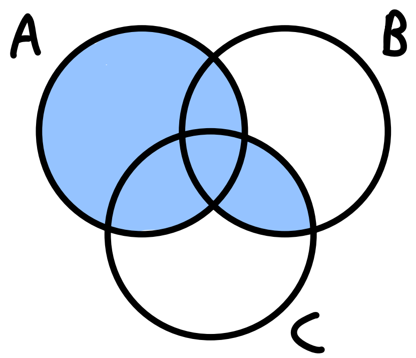
$$A \cup (B \cup C) = (A \cup B) \cup C$$

$$A \cap (B \cap C) = (A \cap B) \cap C$$

7) Distributive laws

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$



8) de Morgan's Laws

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

9) Absorbtion laws

$$A \cup (A \cap B) = A$$

$$A \cap (A \cup B) = A$$

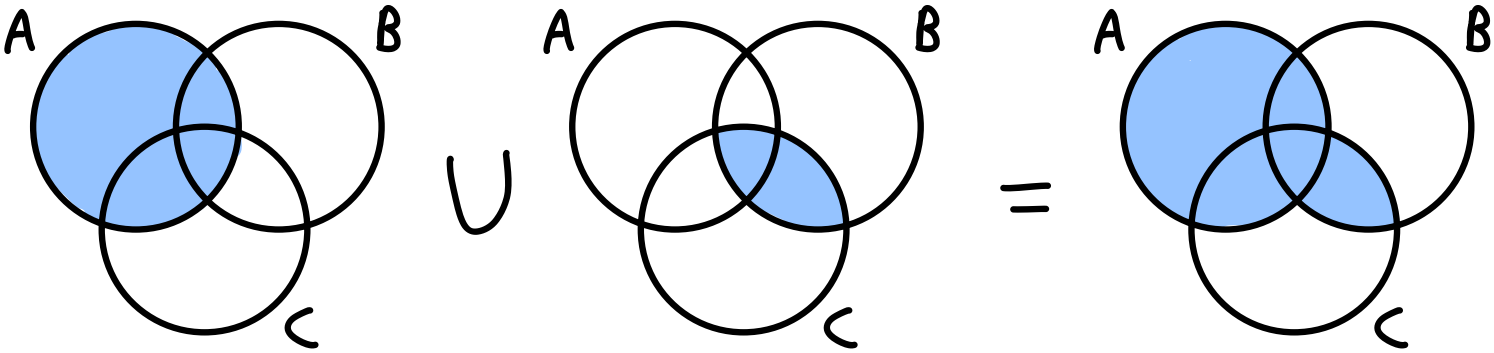
10) Complement laws

$$A \cup \overline{A} = U$$

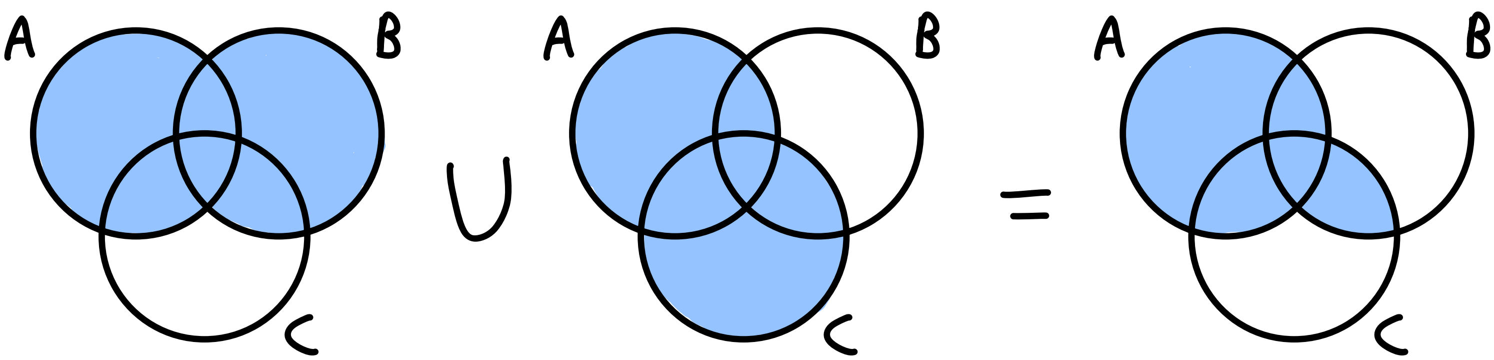
$$A \cap \overline{A} = \emptyset$$

Venn diagram tricks (for intuition only)

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$



A $\quad$ $B \cap C$ $\quad$ $A \cup (B \cap C)$



$A \cup B$ $\quad$ $A \cup C$ $\quad$ $(A \cup B) \cap (A \cup C)$

§2.3: Functions

Def: Let A, B be nonempty sets. A <u>function</u> f from A to B is an assignment of exactly one elt. of B to each elt. of A

Write: $f: A \to B$ $\qquad$ $a \rightsquigarrow \boxed{f} \rightsquigarrow f(a)$

$f(a) \in B$ for $a \in A$

Ex:

a) $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$

b) $f: \mathbb{N} \to \mathbb{N}$, $f(x) = x^2$

not the same function!

c) $A = \{a, b, c\}$ $\quad B = \{x, y, z\}$

$f: A \to B$

$f(a) = x \qquad f(b) = z \qquad f(c) = x$

d) Non-examples

A, B as above

$g(a) = x$
$g(b) = y$
$g(c) = z$
$g(a) = y$

$h(a) = y$
$h(c) = x$

g and h are <u>not</u> functions

Def (cont.):

- A is the <u>domain</u> of f
- B is the <u>codomain</u> of f
- The <u>range/image</u> of f is the set $\{f(a) \mid a \in A\}$
- If $a \in A$, $f(a)$ is the <u>image</u> of a under f
- If $b \in B$, the <u>preimage</u> of b under f is the set

$$f^{-1}(b) = \{a \in A \mid f(a) = b\}$$

Ex: A, B, f as above

f has:

- domain A
- codomain B
- range $\{x, z\}$

The image of c is x

The preimage of x is $\{a, c\}$

The preimage of y is $\emptyset$

Can also do image/preimage of sets

Def: Let $f: A \to B$. Let $C \subseteq A$ and $D \subseteq B$

The image of C is $f(C) = \{f(c) \mid c \in C\}$

The preimage of D is $f^{-1}(D) = \{a \in A \mid f(a) \in D\}$

Ex (cont): $f(\{a, c\}) = \{x\}$

$$f^{-1}(\{x, z\}) = A$$