

Announcements

Final exam: Tuesday, 5/12, 8:00am-11:00am, Everitt 3117

Covers entire course

Policy email to come

Wednesday's class: review (no quiz)

§ 11.2: Applications of trees

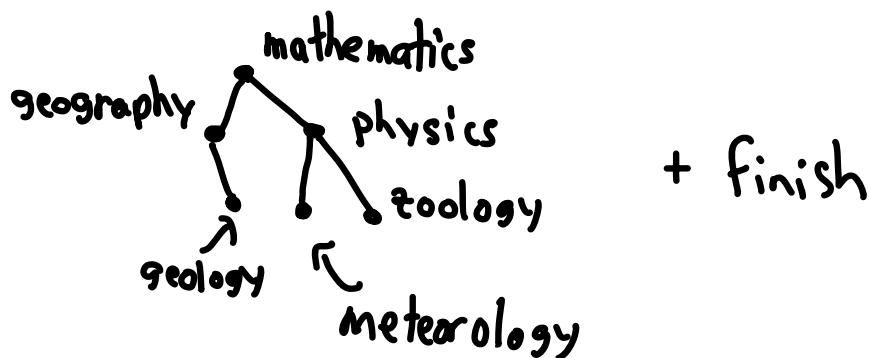
Binary search trees

Suppose we have a list of words, which we want in alphabetical order

Add them to a binary tree, such that

left child < parent < right child

Ex 1: {mathematics, physics, geography, zoology, meteorology, geology, psychology, chemistry}



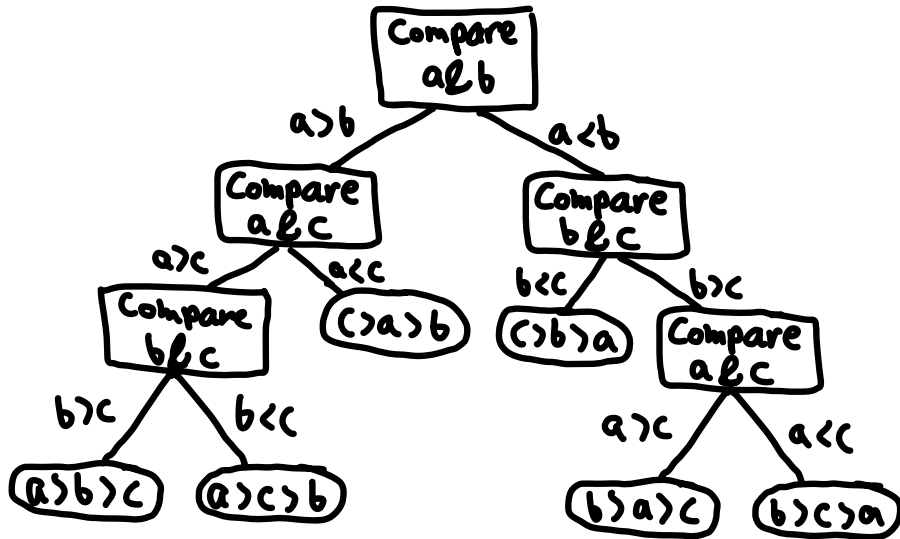
We can use this tree to read off the words in order or to determine whether a word is in the list

(e.g. geology, oceanography)

Decision trees

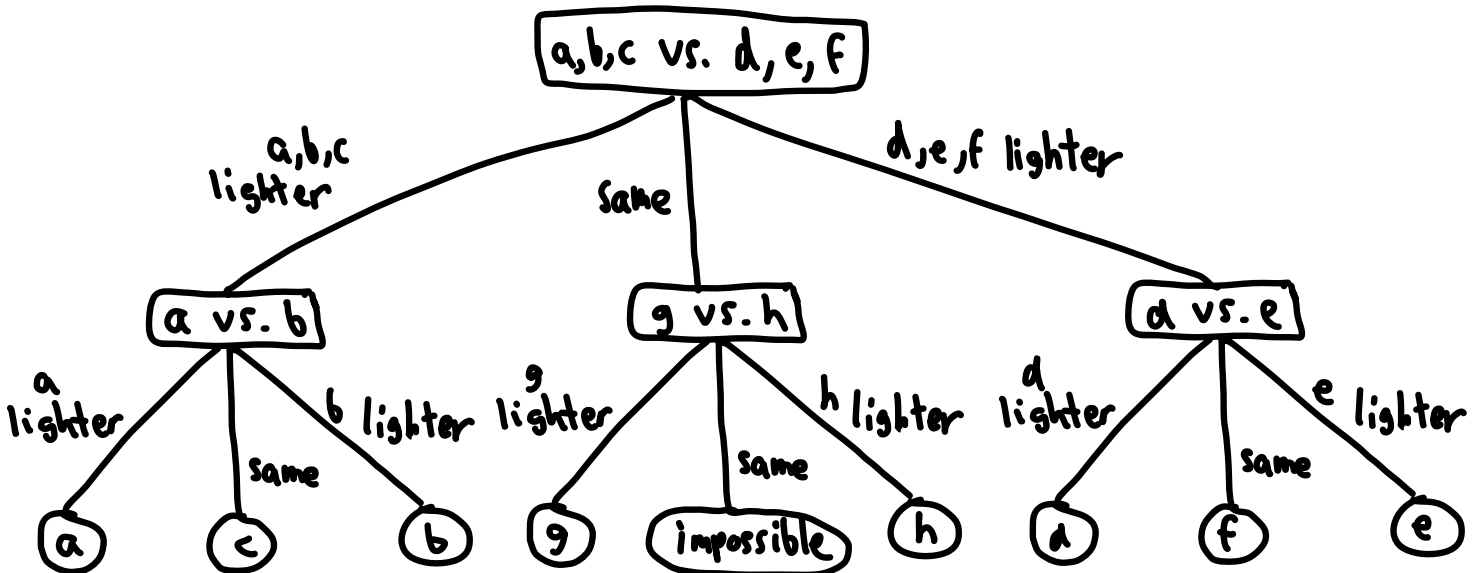
A decision tree is a rooted tree where each internal vertex corresponds to a decision, and this decision moves us to one of the two children. The leaves are the "conclusions"

Ex 4: a, b, c are three distinct numbers. Give a decision tree that orders a, b, c .



Ex 3: Suppose there are 7 coins, all w/ the same weight, and a counterfeit coin that weighs less than the others. How many weighings are needed to determine the counterfeit coin?

Coins: a, b, c, d, e, f, g, h



Game trees

A game tree is a decision tree where the decisions are made at the discretion of 2 or more (alternating) players

Ex 6: Nim:

2 players

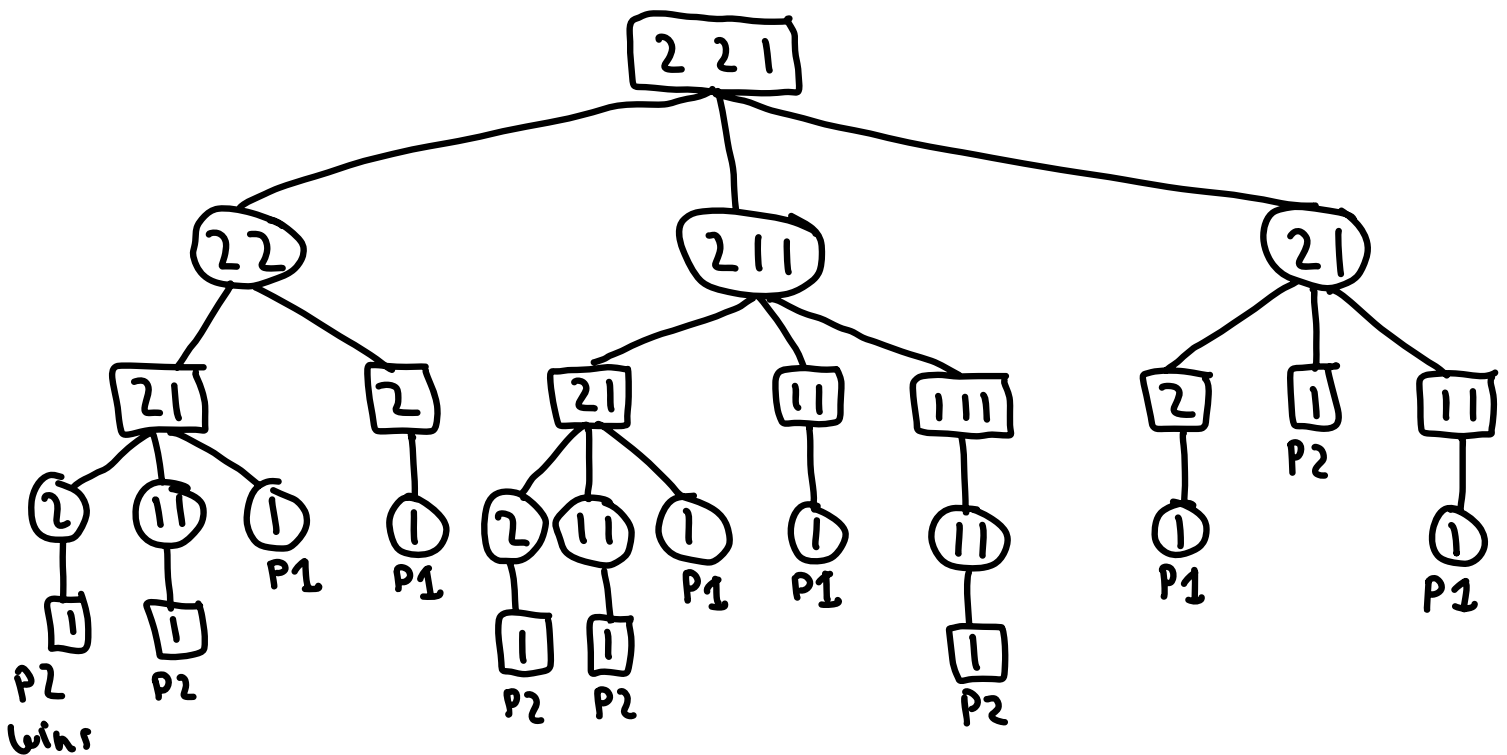
Several piles of stones

Players alternate turns

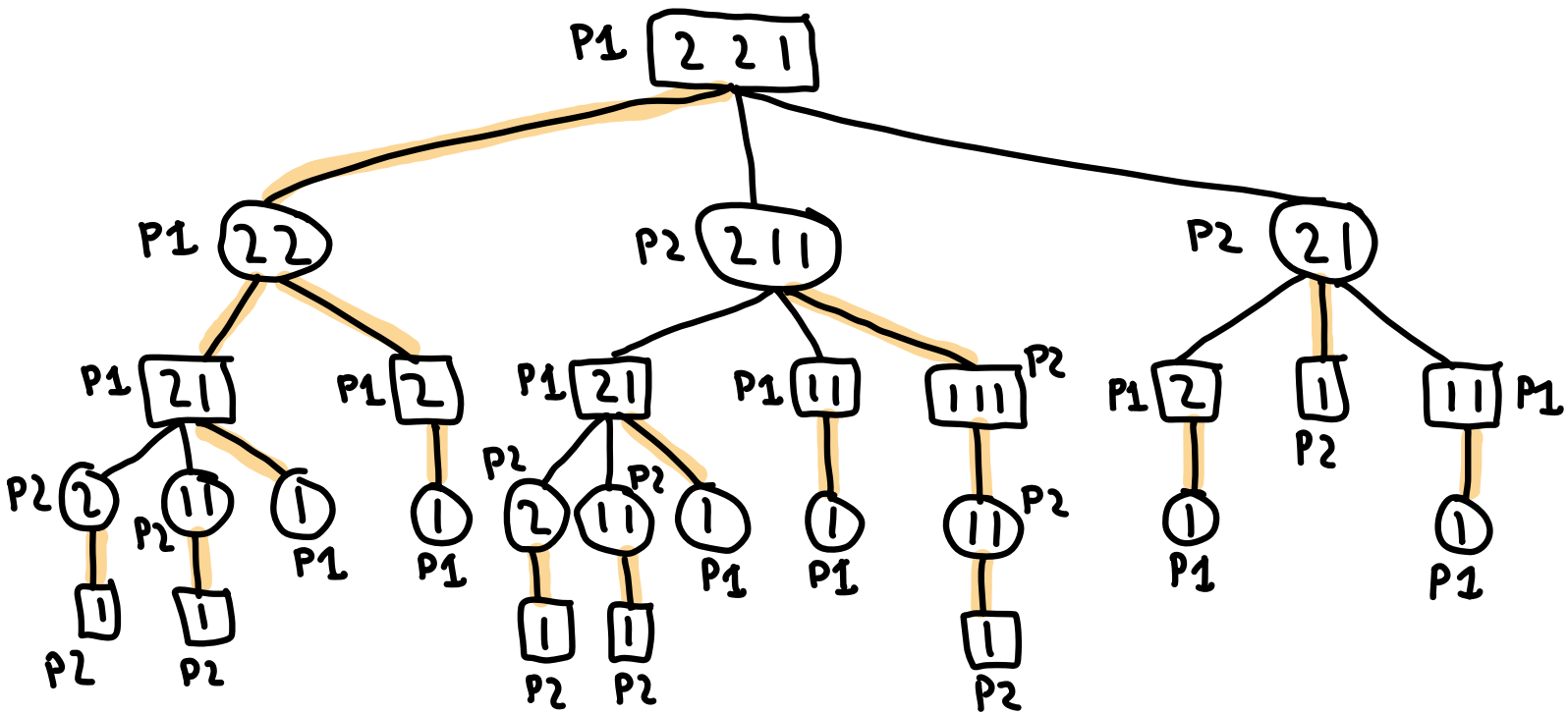
For each turn, a player takes 1 or more stones from one of the piles such that at least one stone remains

The first player who can't move loses!

Start w/ three piles, containing 2, 2, 1 stone(s)



Can use the game tree to see who can force a win, and how



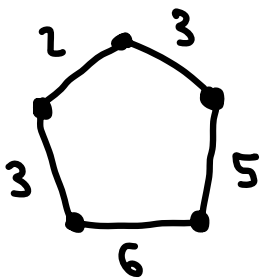
● : best move(s)

§11.4/5: Spanning trees

Def: A spanning tree of a graph G is a subgraph which is a tree and contains all the vertices of G .

If G is a weighted graph, a spanning tree T is minimal if the sum of the edge weights of T is the smallest such value for any spanning tree of G (ties ok).

E.g. C_5

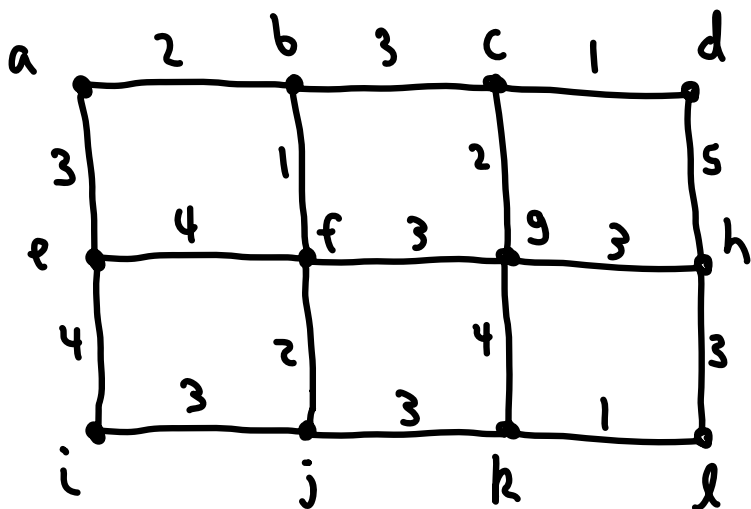


Spanning tree: remove any one edge

Minimal spanning tree:

remove the bottom edge

Class activity: Try to find a minimal spanning tree



We can build up a minimal spanning tree edge by edge:

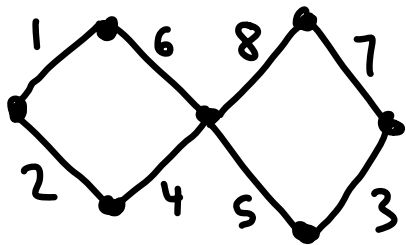
Two algorithms:

Kruskal: Always add an edge w/ smallest wt.
that doesn't create a simple circuit

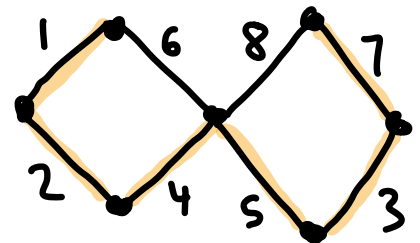
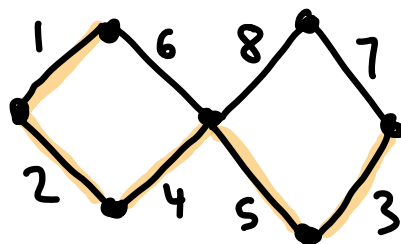
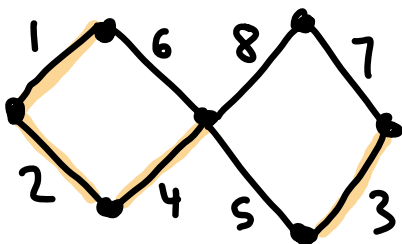
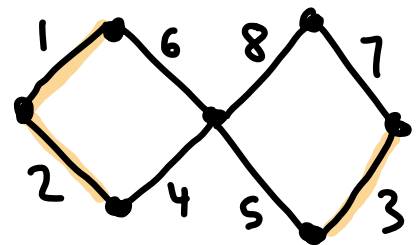
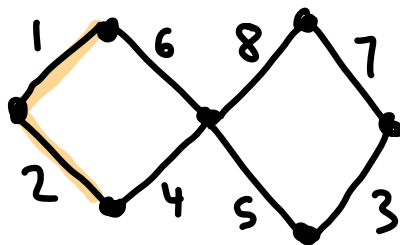
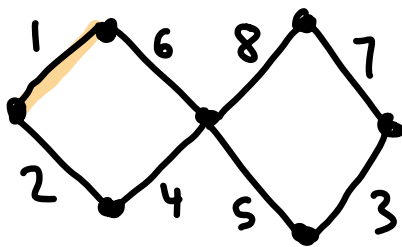
Prim: Always add an edge w/ smallest wt.
that is incident to the built-up part of the spanning
tree and doesn't create a simple circuit

Both algorithms always give a minimal spanning tree

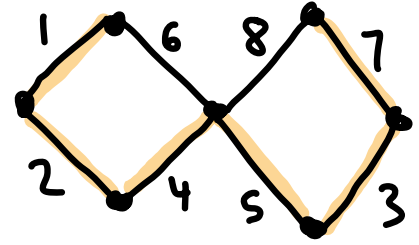
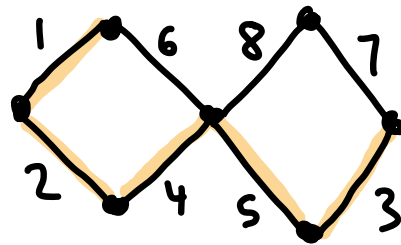
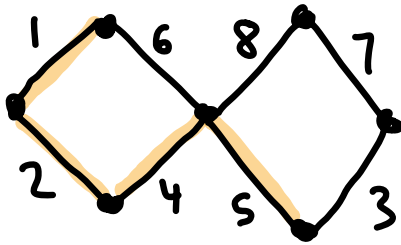
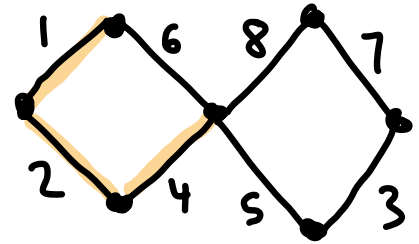
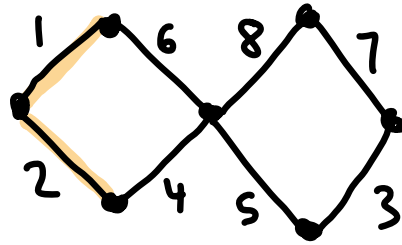
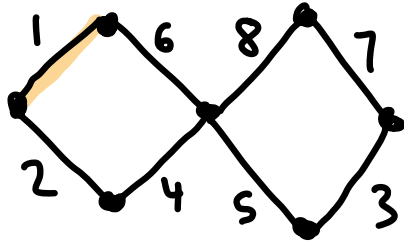
e.g.



Kruskal:



Prim



Notice:

- Prim's expands outward from the first edge, while Kruskal's jumps around
- Both algorithms have to leave off the weight-6 edge in the last step, since it would create a simple circuit.