# Announcements

Midterm 3: Wed. in class

- Covers through Section 11.1
- Reference sheet allowed (one A4 sheet w/ writing on both sides)
- See policy email (practice problems etc.)

No class this Friday (office hour instead)

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# Midterm 3 Review

(Partial) list of topics:

Everything from first two midterms

(sets, functions, algorithms, induction, counting, probability, etc.)

Relations

- Properties: reflexive, symmetric, antisymmetric, transitive
- Operations: complement, inverse, composition
- Representing relations: matrix, digraph
- Equivalence rel'ns
  - Definition
  - Equiv. classes and set partitions

Graphs/digraphs

- Def'ns: simple/multi./nbhd./deg./bipartite/(induced) subgraph
- Handshake thm.
- Special classes of graphs
- Constructions: deletion/contraction/union
- Adjacency & incidence matrices
- Isomorphism
  - Show that graphs <u>are</u> isomorphic: explicit isom., adj. matrices
  - Show that graphs are <u>not</u> isomorphic: different "label-indep. properties"
- Connectivity (for digraphs, weak vs. strong), cut-edges/cut-vertices
- Paths/circuits
  - Eulerian/Hamiltonian (+ criteria for Eulerian)
- Shortest path problems
  - Weighted graphs
  - Dijkstra's algorithm
  - Travelling salesperson
- Planar graphs
  - Direct pf. of planarity/nonplanarity
  - Regions, degree, etc.
  - Euler's formula and consequences
- Graph coloring
  - Maps vs. graphs and their colorings
  - Chromatic number
  - Four-color theorem

Trees

Definitions

Properties

Rooted trees, m-ary trees

Other tips:

Look at HW, quizzes, lecture notes, textbook, other problems

We have lots of def'ns and constructions – learn them and/or write them on your ref. sheet

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Examples:

1) Find the equiv. rel'n corresp. to the following set partition:

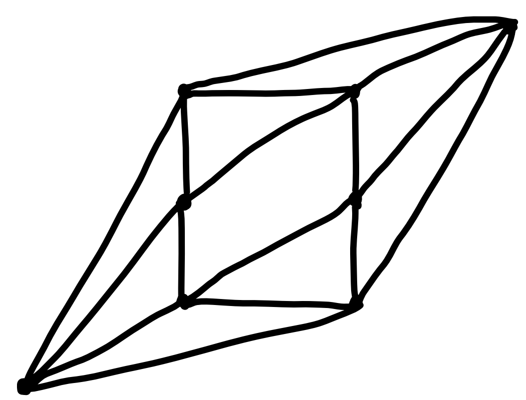
$A_i = \{5k+i \mid k \in \mathbb{Z}\}$

$A = \mathbb{Z} = A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$

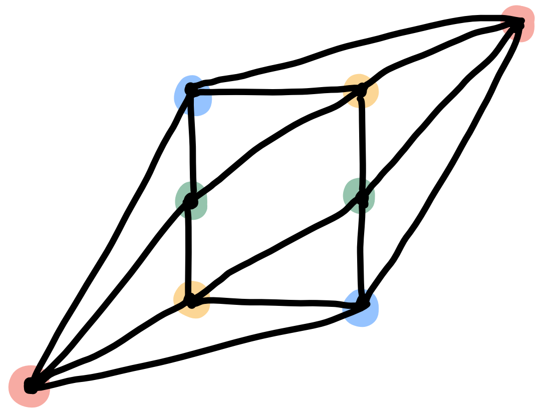
Sol'n: $a \sim b$ if and only if $a-b$ is a mult. of 5

$A_i = [i]$ for $i = 1, 2, 3, 4, 5$

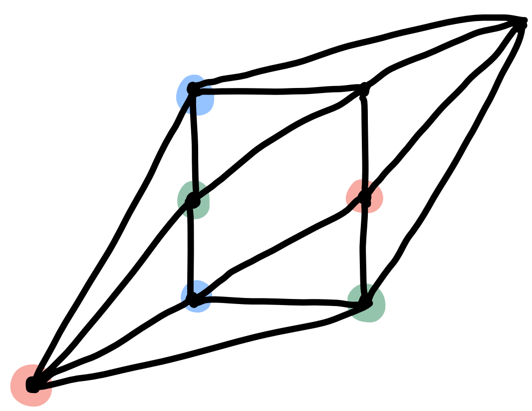
2) Determine the chromatic number of the following graph $G$



Sol'n: G can be 4-colored (see below)



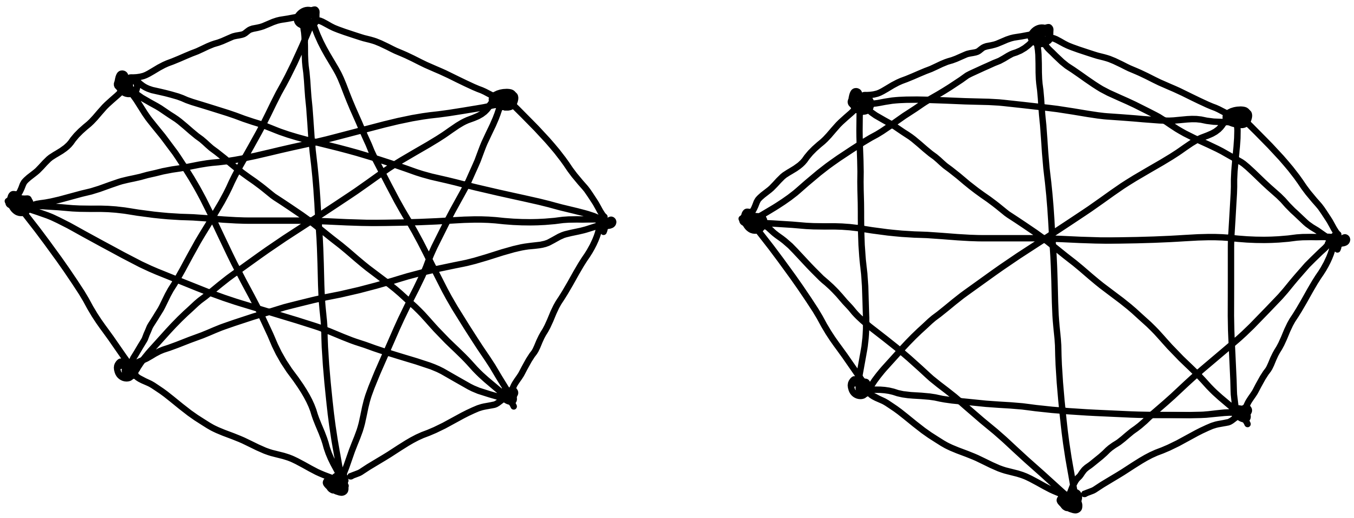
However, no 3-coloring exists. Let red be the color of the bottom-left vertex. The top left vertex must have a different color; let that be blue. The vertex just below it must have a different color; let that be green. Using only those three colors, the following partial coloring is forced:



But then there is no valid color for either of the remaining vertices.

Therefore, $\chi(G) = 4$.

3) (10.3.44) Determine whether or not these graphs are isomorphic.



b) Do they have Eulerian/Hamiltonian paths/circuits?

Sol'n: b) Neither have Eulerian paths/circuits since both have 8 verts. of deg. 5

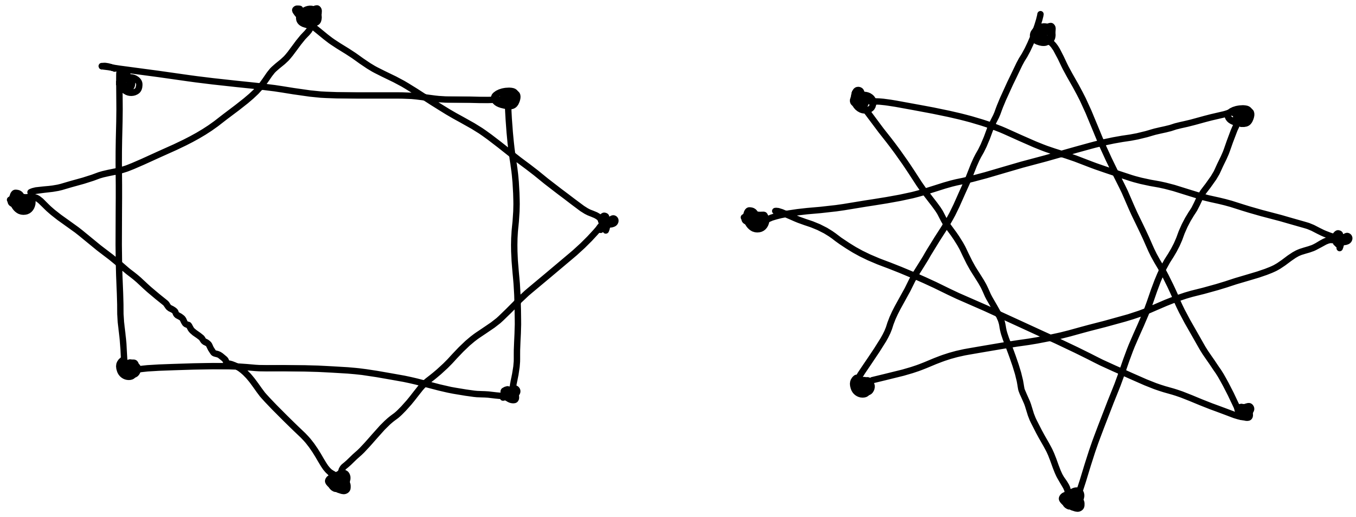
Both have Hamiltonian circuits by traversing the outside edges clockwise

a) Not isomorphic.

Several options to show this:

- In both graphs, each vertex has exactly two nonneighbors. Take the nonneighbors of these two vertices; in the left graph, they are the same while in the right graph, they are different.

• Take the complements:



Right is conn., left is disconn., so they are not isom.

+ general discussion