Announcements

Midterm 3: Wed 4/29 in class

Practice problem solns posted

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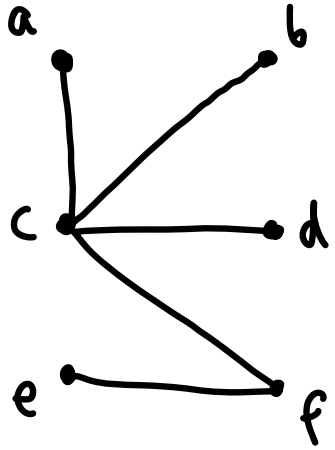
§11.1: Trees

Def: A tree is a conn. (undir.) graph w/ no simple circuits

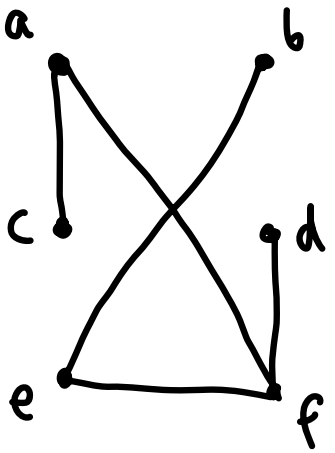
A forest is a graph consisting of one or more trees

Class activity: Tree or no tree?

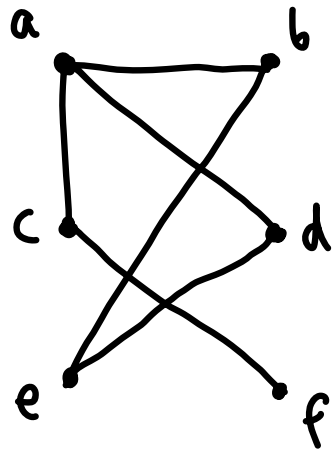
a)



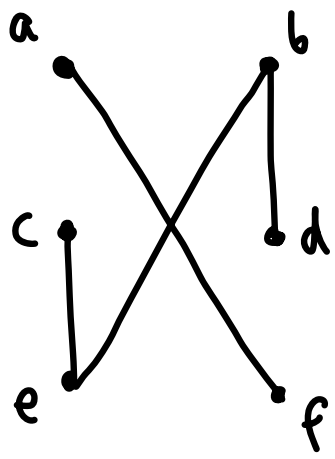
b)



c)



d)



Properties: Suppose T is a tree.

a) T has no loops or mult. edges

b) There is exactly one simple path btwn. any two verts.

c) $|E| = |V| - 1$

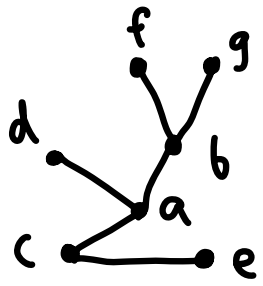
d) Every edge is a cut edge

e) Adding any edge creates a simple circuit

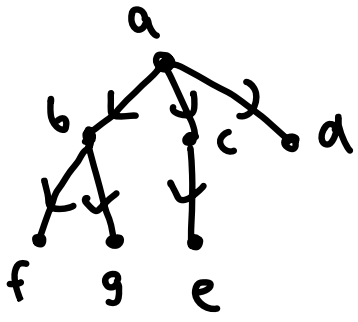
Def: A rooted tree is a tree in which one vertex has been designated the root

Note: We sometimes think of a rooted tree as a digraph where every edge is directed away from the root

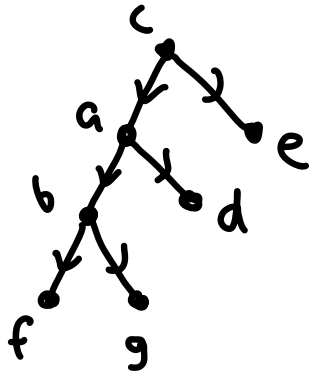
Ex:



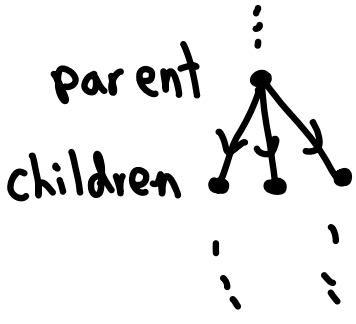
tree

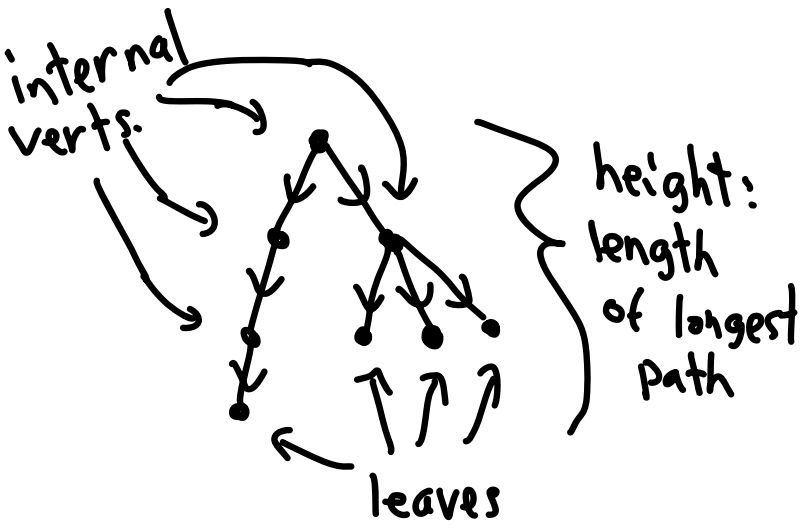


rooted tree
w/ root a



rooted tree
w/ root c





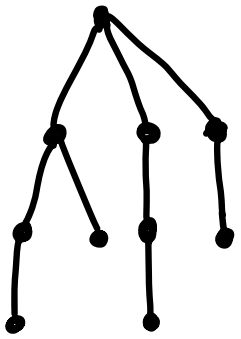
Def: A rooted tree is called an <u>m-ary tree</u>
if every internal vertex has $\leq m$ children.
If every internal vertex has $= m$ children, it is called <u>full</u>

An m-ary tree w/ $m = 2$ is a <u>binary tree</u>

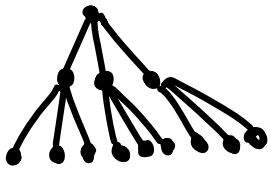
An m-ary tree of height h has $\leq m^h$ leaves

Class activity: Count the number of leaves, internal vertices, (total) vertices in

a)



b) A full 3-ary tree of height 2



b) A full binary tree of height 3

d) A full binary tree of height 7

Facts:

a) A full binary tree of height h has $2^{h+1} - 1$ vertices, 2^h of which are leaves, and $2^h - 1$ of which are internal vertices.

b) A full m-ary tree of height h has $1 + m + m^2 + \cdots + m^h$ verts. m^h of which are leaves, and $1 + m + \cdots + m^{h-1}$ of which are internal vertices.

c) A full m-ary tree w/ i int. verts. has $mi + 1$ verts. and $(m-1)i + 1$ leaves

d) A full m-ary tree w/ n verts. has $\frac{n-1}{m}$ int. verts. and $\frac{(m-1)n + 1}{m}$ leaves

§11.2: Applications

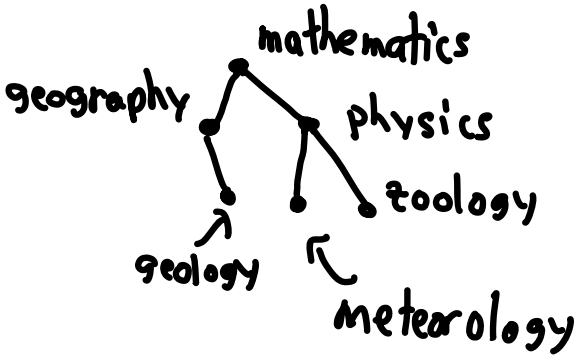
Binary search trees

Suppose we have a list of words, which we want in alphabetical order

Add them to a binary tree, such that

left child < parent < right child

Ex 1: {mathematics, physics, geography, zoology, meteorology, geology, psychology, chemistry}



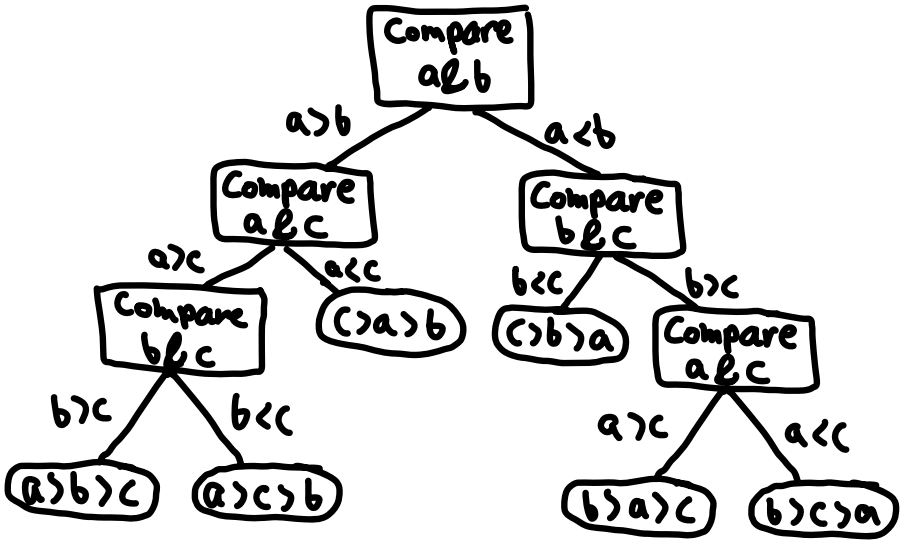
Class activity finish this tree.

We can use this tree to read off the words in order or to determine whether a word is in the list (e.g. geology, oceanography)

Decision trees

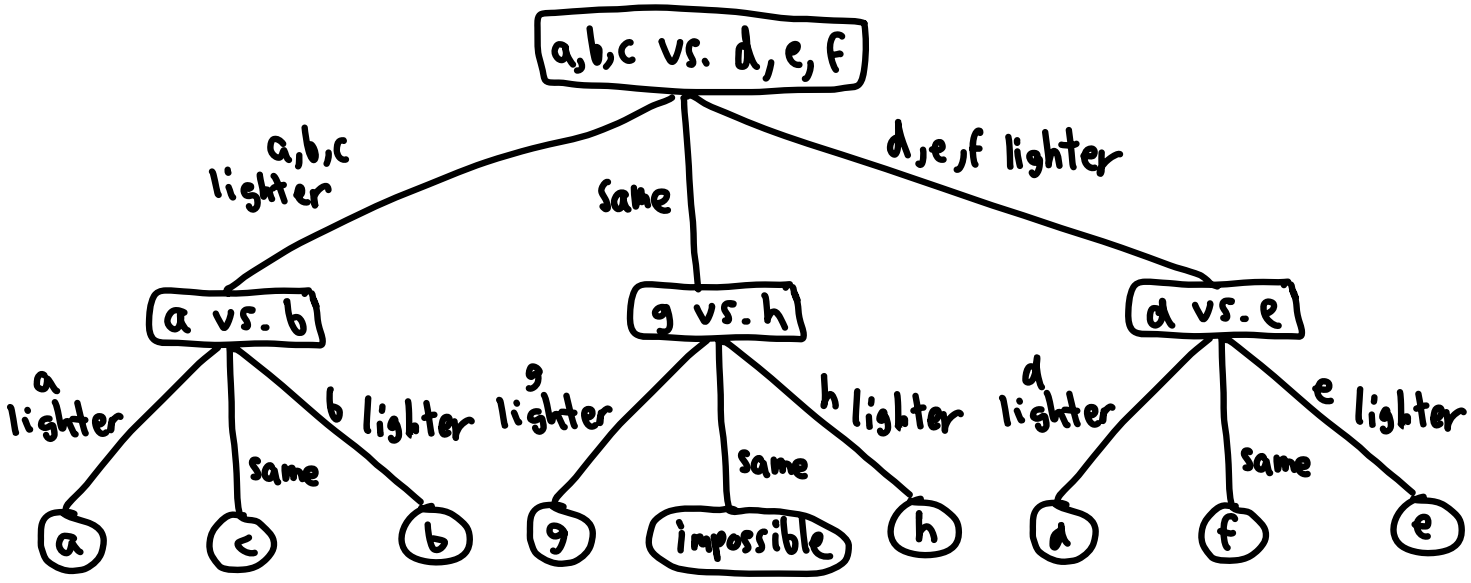
A decision tree is a rooted tree where each internal vertex corresponds to a decision, and this decision moves us to one of the two children. The leaves are the "conclusions"

Ex 4: a, b, c are three distinct numbers. Give a decision tree that orders a, b, c.



Ex 3: Suppose there are 7 coints, all w/ the same weight, and a counterfeit coin that weighs less than the others. How many weighings are needed to determine the counterfeit coin?

Coins: a, b, c, d, e, f, g, h



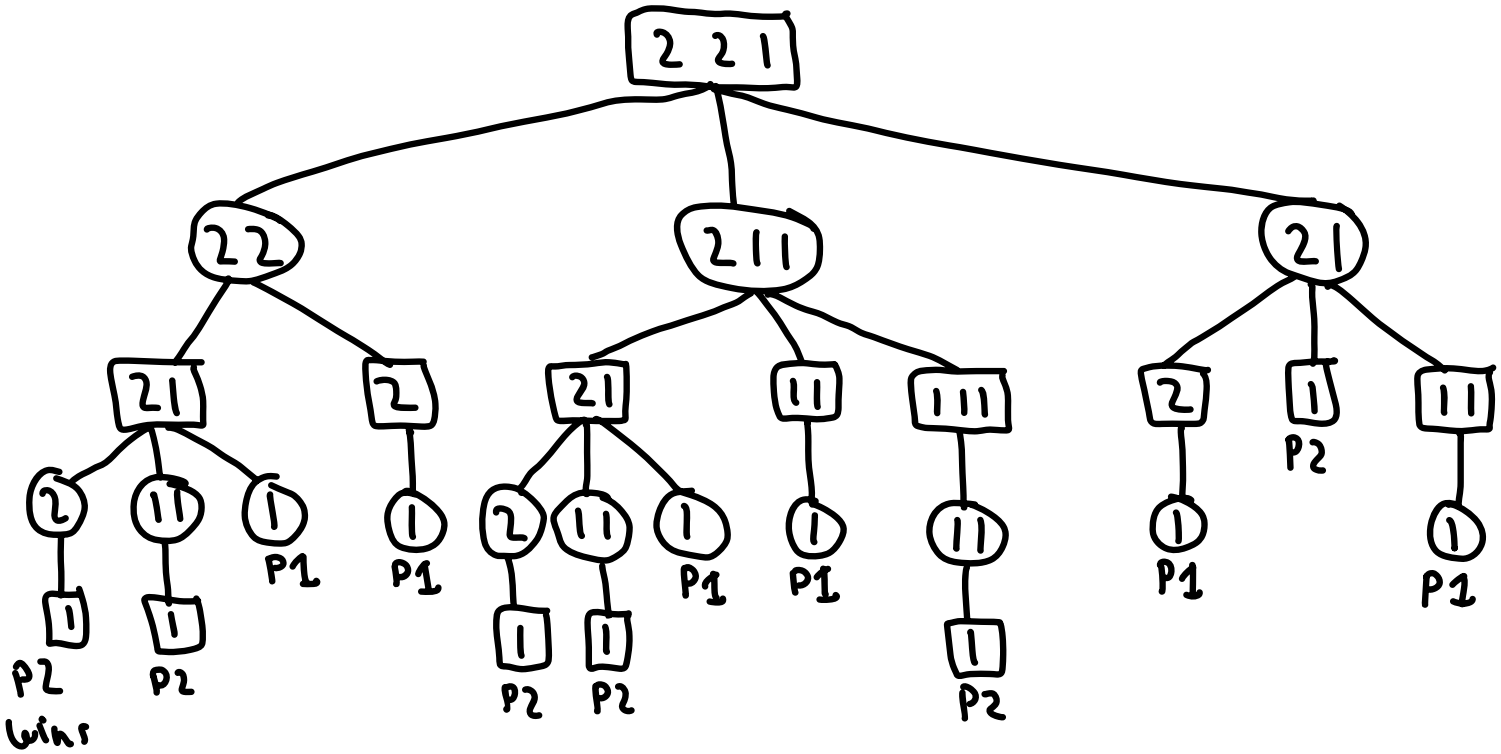
Game trees

A game tree is a decision tree where the decisions are made at the discretion of 2 or more (alternating) players

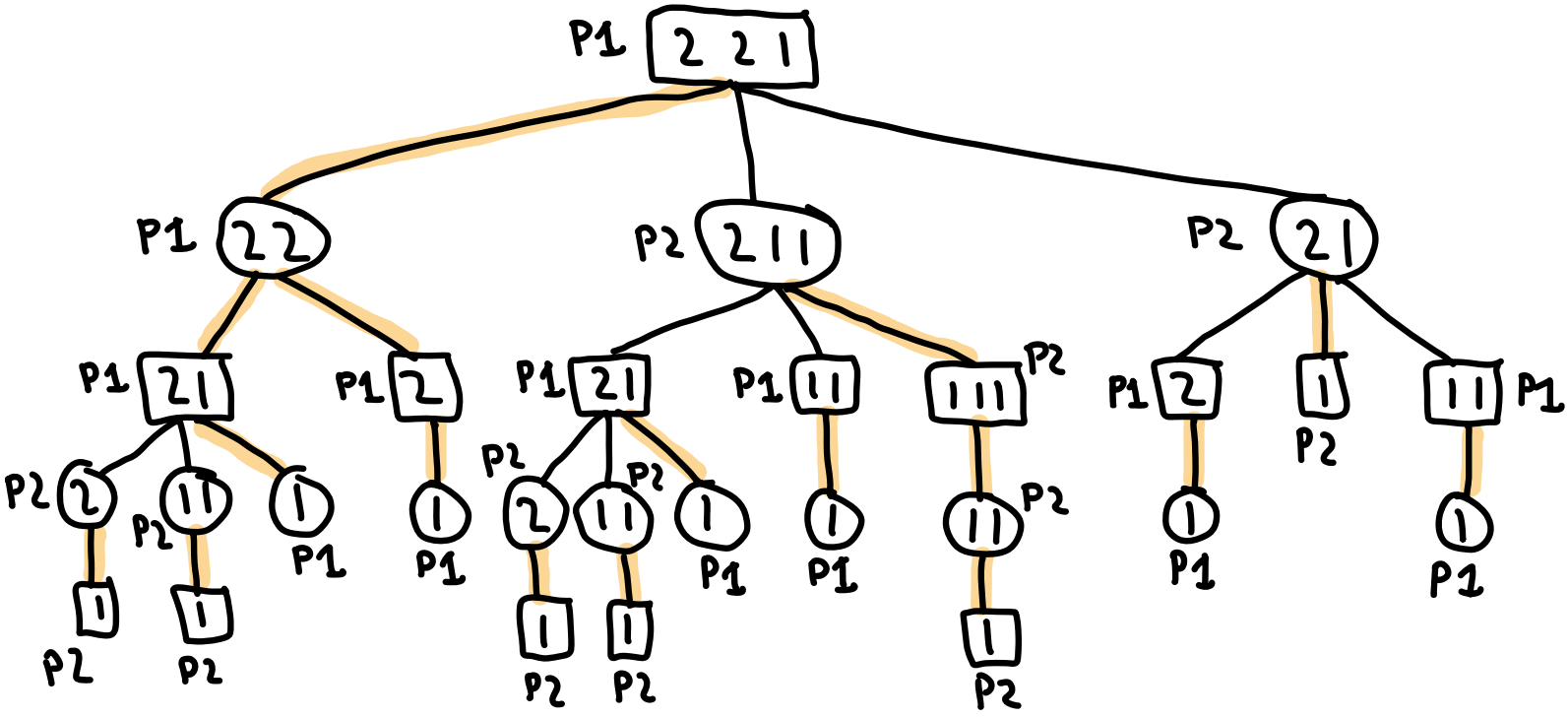
Ex 6: Nim:

2 players
Several piles of stones
Players alternate turns
For each turn, a player takes 1 or more stones from one of the piles such that at least one stone remains
The first player who can't move loses!

Start w/ three piles, containing 2, 2, 1 stone(s)



Can use the game tree to see who can force a win, and how


P1 2 2 1
P1 2 2
P2 2 1 1
P2 2 1
P1 2 1
P1 2
P1 2 1
P1 1
1 1 1
P2
P1 2
1
P2
1 1
P1
P2 2 1 1 1