Recall: An Eulerian path/circuit uses each edge exactly once

A Hamiltonian path/circuit uses each vertex exactly once (except, if circuit, $v_0 = v_n$)

Theorem:

a) A conn. graph has an Eulerian circuit if and only if all degrees are even

b) A conn. graph has an Eulerian path if and only if ≤ 2 degrees are odd

c) A weakly conn. digraph has an Eulerian circuit if and only if $\deg^-(v) = \deg^+(v)$ for all vertices v

d) A weakly conn. digraph has an Eulerian path if and only if $\deg^-(v) = \deg^+(v)$ for all vertices v except for at most two, one of which has $\deg^-(v) = \deg^+(v) + 1$, and the other of which has $\deg^-(v) = \deg^+(v) - 1$

Which graphs have Hamiltonian paths/circuits?

Many classes of graphs do e.g. K_n, C_n, W_n, Q_n

But in general the problem is very hard (NP-complete!)

§10.6: Shortest-path problems

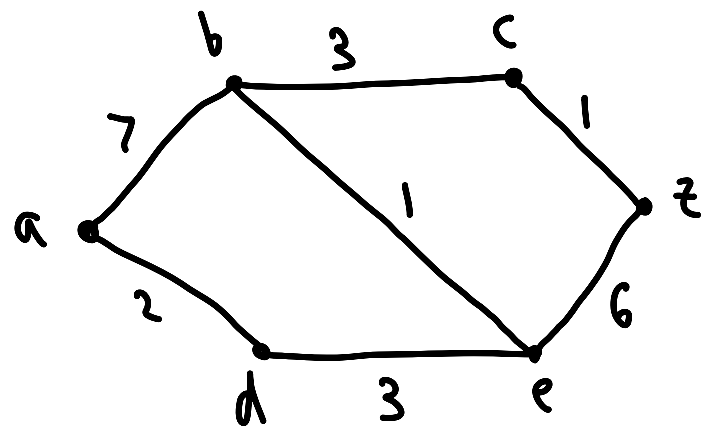
For this section, $G = (V, E)$ is a simple conn. (undir.) graph

Each edge $e \in E$ has a <u>weight</u> $wt(e)$ (always a pos. num.)

Question: Given two vertices, what is the <u>shortest path</u> from one to the other?

add up the wt of each edge

Ex:



Shortest path from a to z?

Some possibilities:

a, b, c, z : $7+3+1 = 11$

a, d, e, z : $2+3+6 = 11$

a, b, e, z : $7+1+6 = 14$

a, d, e, b, c, z : $2+3+1+3+1 = 10$ ✓ shortest path

Note also that the shortest path from a to b is a, d, e, b

Dijkstra's algorithm

Input: weighted conn. simple graph $G=(V,E)$ (all wts. pos.)
start vertex $a \in G$
$w(u,v)$ = length of the edge btwn. u and v (∞ if no such edge)

Output: shortest path distance from a to all other vertices

Algorithm:

$L(v_i)$: distance from a to v_i

S: vertices considered so far

$L(a) := 0$
$L(v) := \infty$ for all other vertices v

$S = \emptyset$

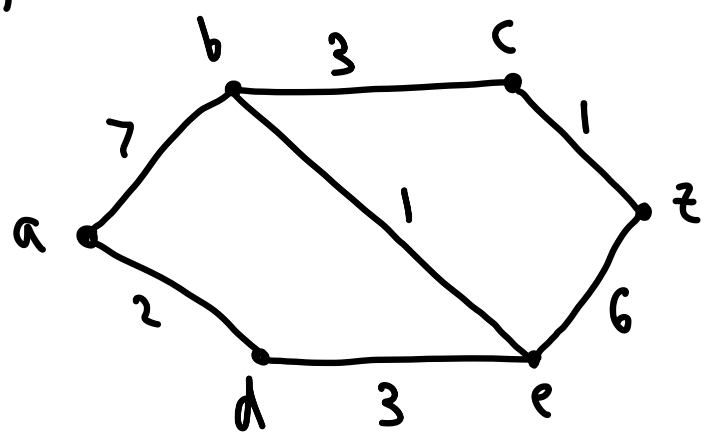
while $S \neq V$
  $u :=$ a vertex not in S with $L(u)$ minimal
  Add u to S
  for all vertices v not in S
    if $L(u) + w(u,v) < L(v)$,
      then $L(v) := L(u) + w(u,v)$

i.e. passing through u gives a shorter path to v.

return $\{(v, L(v)) \mid v \in V\}$

distances from a to v for all vertices v

Ex (cont.):



$L(a) = 0$ $\quad$ $L(d) = \infty$

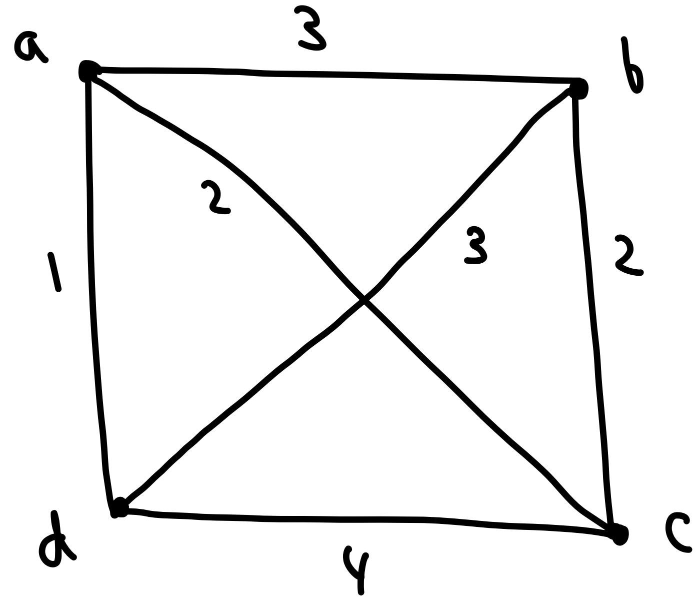
$L(b) = \infty$ $\quad$ $L(e) = \infty$ $\quad$ $S = \emptyset$

$L(c) = \infty$ $\quad$ $L(z) = \infty$

Travelling salesperson problem: Let $G = K_n$ (weighted graph)

Question: What is the shortest Hamiltonian circuit of G?

Ex:



Starting w/ a, there are 6 Hamiltonian circuits

a, b, c, d, a : $3+2+4+1=10$

a, b, d, c, a

a, c, b, d, a

a, c, d, b, a

a, d, b, c, a

a, d, c, b, a

(If time:)

Class activity!

finish this example