

Quiz 5 today!

HW10 posted (due Wed. 4/22)

Recall: A path is an alternating sequence

$$v_0, e_1, v_1, e_2, \dots, e_n, v_n$$

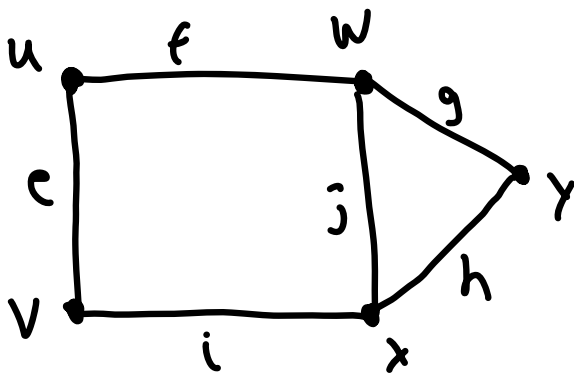
where  $v_i \in V$ ,  $e_i \in E$ ,  $e_i$  has endpoints  $v_{i-1}$  and  $v_i$

Simple path: no repeated edges

Circuit:  $v_0 = v_n$

Simple graph, don't need to write the edges

Warm-up:



Which of the following are paths? Circuits?

Simple paths/circuits?

a)  $u, f, w, j, x, i, v, e, u$

d)  $u, w, x, y, w, x, y$

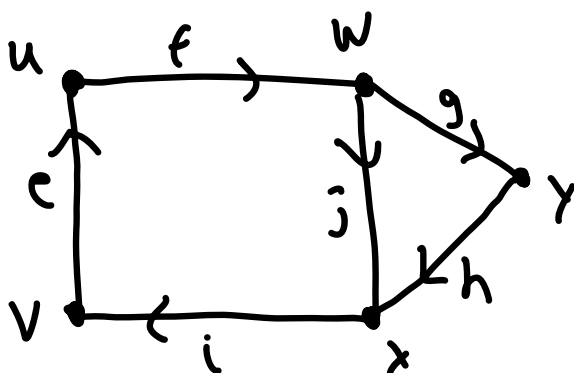
b)  $u, w, x, v, u$

e)  $u, f, w, f, u, f, w$

c)  $u, e, w, i, x, j, v, f, u$

Def: Paths/circuits are the same in digraphs except they must follow the arrow

Class activity:



Which of the following are paths? Circuits?

Simple paths/circuits?

a)  $u, f, w, j, x, i, v, e, u$

d)  $u, w, x, y, w, x, y$

b)  $u, w, x, v, u$

e)  $u, f, w, f, u, f, w$

c)  $u, e, w, i, x, j, v, f, u$

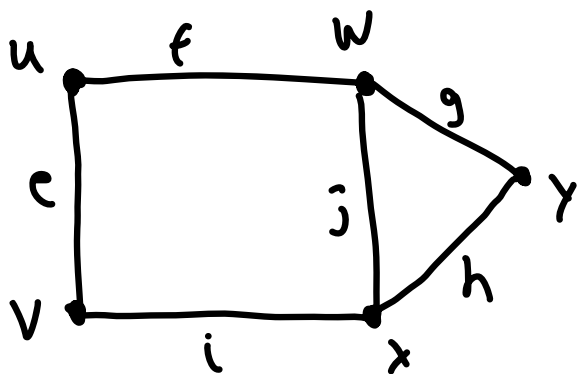
Def: a) A graph is connected if there is a path from every vertex to every other vertex

b) A digraph is strongly connected if there is a path from every vertex to every other vertex

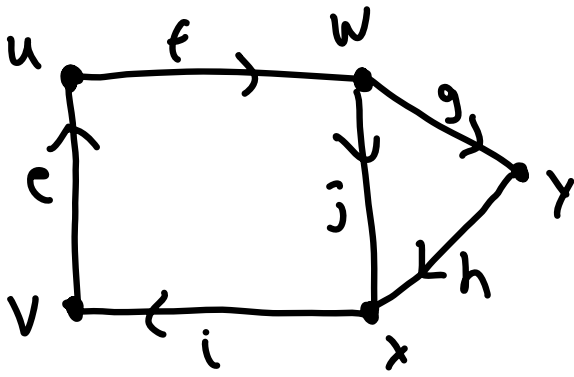
c) A digraph is weakly connected if there is a path from every vertex to every other vertex in the underlying graph (erase the arrows)

d) A cut edge / cut vertex in a graph is an edge / vertex which, when deleted, disconnects the graph

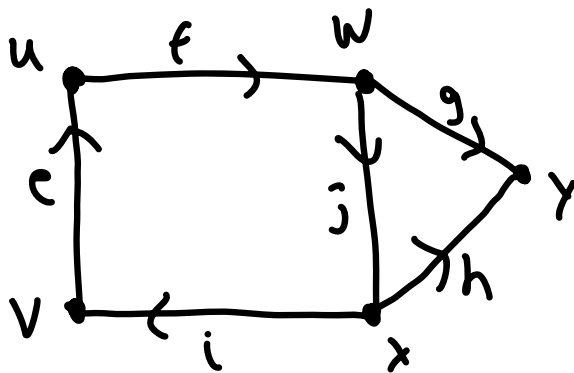
Examples:



Connected, no  
cut-edges/vertices



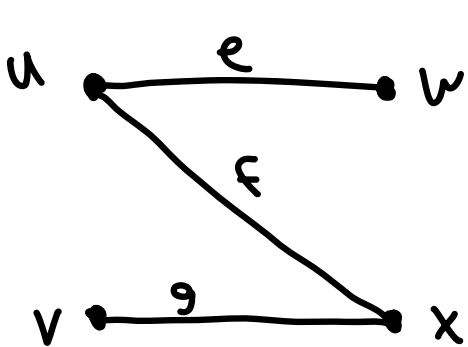
strongly  
conn.



weakly conn.  
since no path from  
y to e.g. v



disconnected

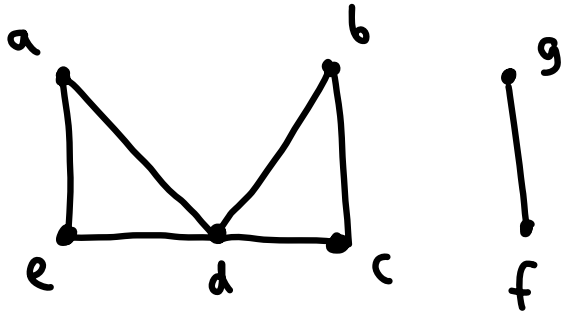


connected  
cut-vertices: u, x  
cut-edges: e, f, g

Def: An Eulerian path/circuit uses each edge exactly once

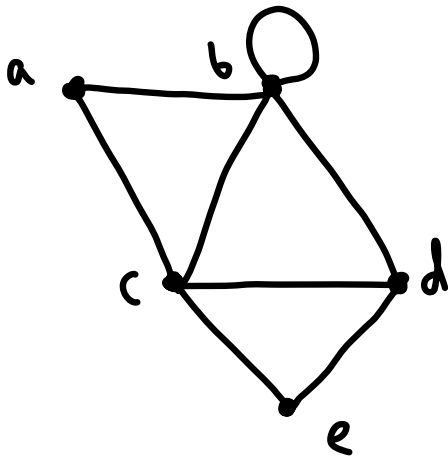
A Hamiltonian path/circuit uses each vertex exactly once  
(except, if circuit,  $v_0 = v_n$ )

Ex: (i)

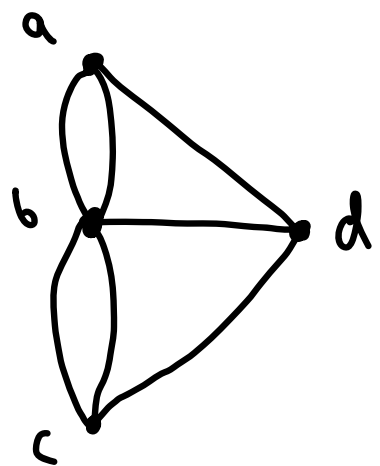


(So let's only deal w/ conn. graphs)

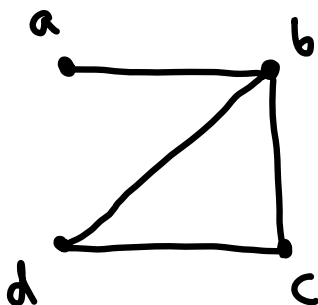
(ii)



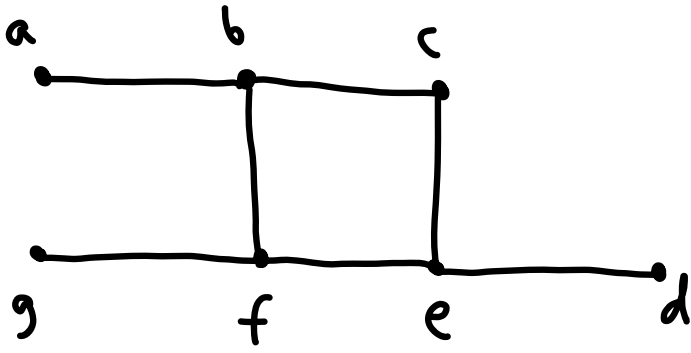
(iii)



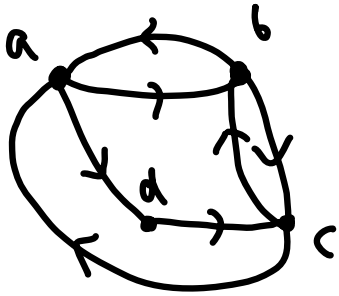
(iv)



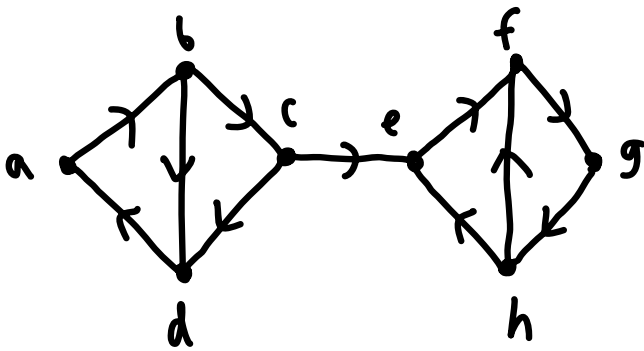
v)



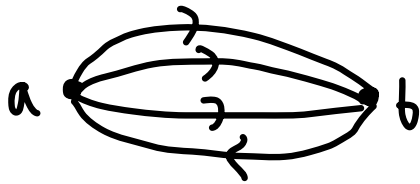
vi)



vii)



viii)



Theorem:

- a) A conn. graph has an Eulerian circuit if and only if all degrees are even
- b) A conn. graph has an Eulerian path if and only if  $\leq 2$  degrees are odd
- c) A weakly conn. digraph has an Eulerian circuit if and only if  $\deg^-(v) = \deg^+(v)$  for all vertices  $v$
- d) A weakly conn. digraph has an Eulerian path if and only if  $\deg^-(v) = \deg^+(v)$  for all vertices  $v$  except for at most two, one of which has  $\deg^-(v) = \deg^+(v) + 1$ , and the other of which has  $\deg^-(v) = \deg^+(v) - 1$

Which graphs have Hamiltonian paths/circuits?

Many classes of graphs do e.g.  $K_n$ ,  $C_n$ ,  $W_n$ ,  $Q_n$

But in general the problem is very hard (NP-complete!)