Announcements

Quiz 5: this Wed.

Midterm 3 Wed. 4/29 in class (2 weeks +)

Policy email coming soon

Midterm 2 grades released + solns posted

Mean: 64.2/78
Median: 64/78
Std. dev.: 7.7

Q1: 69%
Q2: 81%
Q3: 94%
Q4: 100%
Q5: 90%

Gradelines:

A/A-: 66-78
B+/B/B-: 54-65.9
C+/C/C-: 40-53.9
D+/D/D-: 18-39.9

} out of 78

Gradeline calculator updated

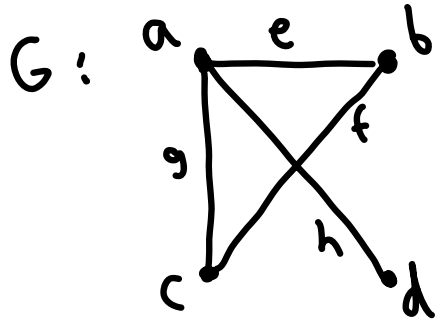
Regrade requests open for 1 week

Def: Let G be a graph w/ vertices $v_1, \dots, v_n$.
and edges $e_1, \dots, e_m$

The incidence matrix of G is the
matrix $\text{Inc}_G = [m_{ij}]$

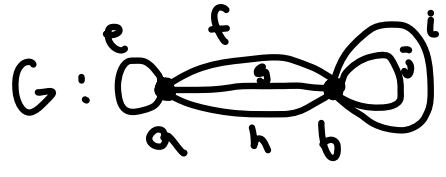
where $m_{ij} = \begin{cases} 1, \text{ if } v_i \text{ is an endpoint (or both endpoints!) of } e_j \\ 0, \text{ otherwise} \end{cases}$

Ex:



$$\text{Inc}_G = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} e & f & g & h \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Ex:



$$\text{Inc}_G = \begin{array}{c} \\ a \\ b \end{array} \begin{array}{c} \begin{array}{cccccc} e & f & g & h & i & j \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

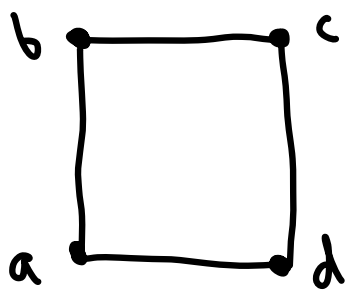
Def: Let $G = (V_1, E_1)$ and $H = (V_2, E_2)$ be simple graphs. A function $f: V_1 \to V_2$ is an <u>isomorphism</u> if

a) f is a bijection

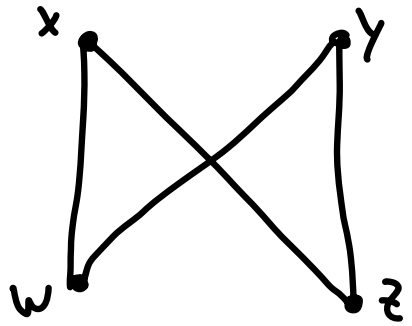
b) $f(a)$ and $f(b)$ are adj. if and only if a and b are adj.

If any isomorphism exists, G and H are <u>isomorphic</u>

Ex 8:



G



H

Graph isomorphism:

$f(a) = w$ $\quad$ $f(c) = z$

$f(b) = x$ $\quad$ $f(d) = y$

Isomorphic graphs have to have the same:

a) number of vertices

b) number of edges

c) lists of degrees

So if G & H differ on any of these $\rightsquigarrow$ not isomorphic!

Be careful:

same a), b), c), but not isomorphic!

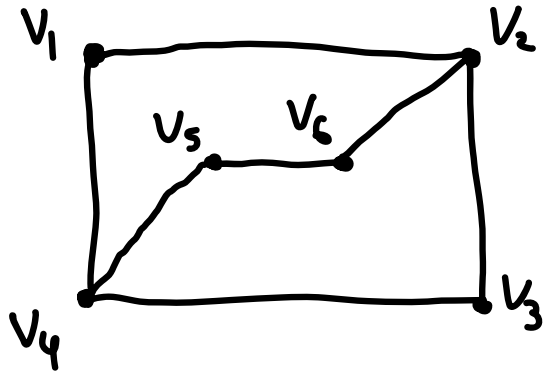
Two ways to show two graphs <u>are</u> isom.

1) Find an isomorphism

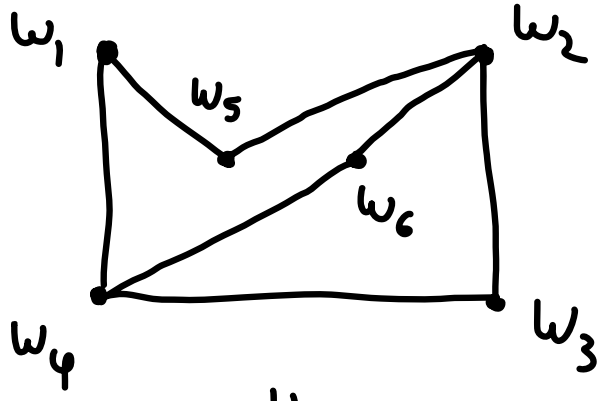
2) Show that the adjacency matrices are the same for some ordering of the vertices

(always same ordering on rows & cols!)

Ex 11:



G



H

$$Adj_G = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$Adj_H = \begin{array}{c} \\ w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{array} \begin{array}{c} \begin{array}{cccccc} w_1 & w_2 & w_3 & w_4 & w_5 & w_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Not the same

But... put the vertices in a different order, and

	w_6	w_2	w_3	w_4	w_1	w_5
w_6	0	1	0	1	0	0
w_2	1	0	1	0	0	1
w_3	0	1	0	1	0	0
w_4	1	0	1	0	0	0
w_1	0	0	0	1	0	1
w_5	0	1	0	0	1	0

So G and H are isomorphic.

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§10.4: Connectivity

Def: A <u>path</u> is an alternating sequence

$$v_0, e_1, v_1, e_2, \ldots, e_n, v_n$$

$v_i \in V$, $e_i \in E$ e_i has endpoints v_{i-1} & v_i

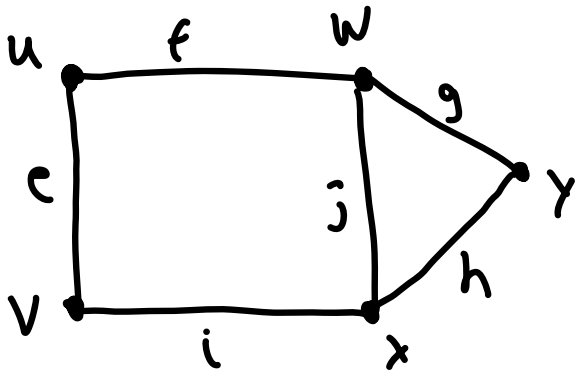
If no repeated edges, the path is <u>simple</u>

If $v_0 = v_n$, it is a <u>circuit</u>

If G is a simple graph, don't need to write the edges, since they're implies

* Note: terminology matches Rosen, different from other sources

Class activity:



Which of the following are paths? Circuits? Simple paths/circuits?

a) u, f, w, j, x, i, v, e, u

b) u, w, x, v, u

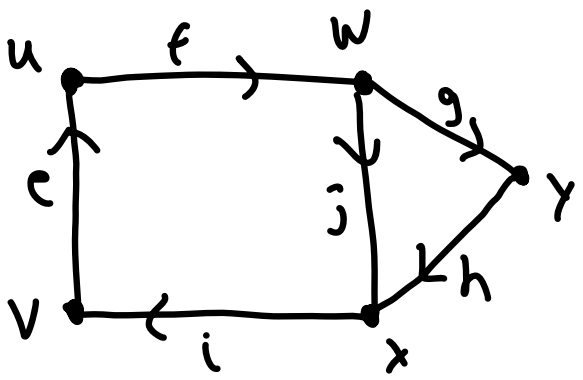
c) u, e, w, i, x, j, v, f, u

d) u, w, x, y, w, x, y

e) u, f, w, f, u, f, w

Def: Paths/circuits are the same in digraphs except they must <u>follow the arrow</u>

Class activity:



Which of the following are paths? Circuits? Simple paths/circuits?

a) u, f, w, j, x, i, v, e, u

b) u, w, x, v, u

c) u, e, w, i, x, j, v, f, u

d) u, w, x, y, w, x, y

e) u, f, w, f, u, f, w

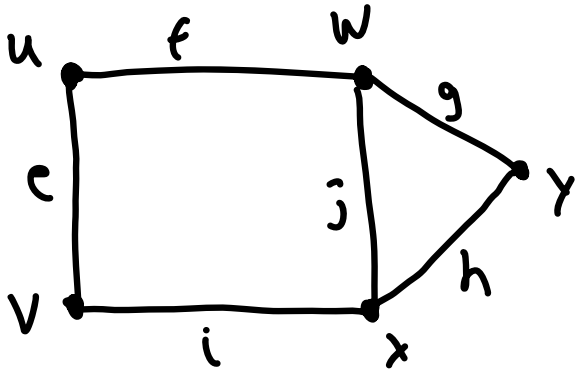
Def: a) A graph is connected if there is a path from every vertex to every other vertex

b) A digraph is strongly connected if there is a path from every vertex to every other vertex

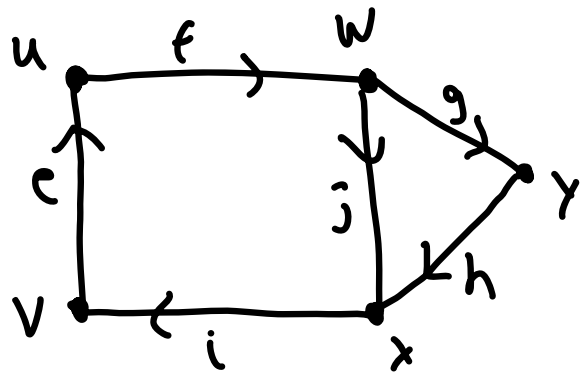
c) A digraph is weakly connected if there is a path from every vertex to every other vertex in the underlying graph (erase the arrows)

d) A cut edge/cut vertex in a graph is an edge/vertex which, when deleted, disconnects the graph

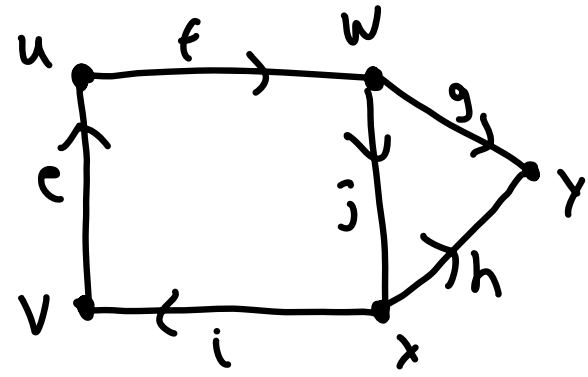
Examples:



Connected, no cut-edges/vertices

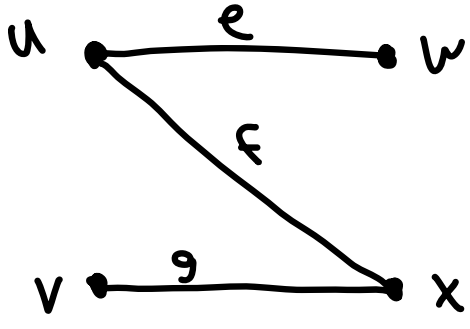


strongly conn.



weakly conn.
since no path from y to e.g. v

disconnected



connected
cut-vertices: u, x
cut-edges: e, f, g