

Announcements

Midterm 1 grades released + solns posted

Mean: 65.2/86

Median: 64/86

Std. dev.:

Q1: 92%

Q2: 82%

Q3: 81%

Q4: 87%

Q5: 46%

Gradelines:

A/A-: 70-85

B+/B/B-: 55-69.9

C+/C/C-: 38-54.9

D+/D/D-: 15-37.9

} out of 86

Gradeline
calculator
posted

Regrade requests open for 1 week

No quiz today

§6.5: Generalized Permutation and Combinations

$P(n, k)$ and $\binom{n}{k}$ refer to permutations/combinations
without repetition and with distinguishable objects

e.g. ABCDEF

BCAE

4-permutation

$\{C, E\}$

2-combination

If we allow repetition, we allow examples like

BBBB

4-perm
w/rep.

$\{C, C\}$

2-comb.
w/rep.

For a set of size n , the number of

r -perms w/
repetition is

$$n^r$$

(by prod. rule)

r -combs w/
repetition is

$$\binom{n+r-1}{n} (*)$$

Idea behind (*): "sticks and stones".

Stones are the elements

Sticks are the "separators"

Ex 4: 4 different kinds of cookie. How many different ways are there to choose 6 cookies with (potential) repetition?

e.g.

* * | * | | * * *

2 of type 1 1 of type 2 0 of type 3 3 of type 4

6 stars (rookies)

4 types, so $4-1=3$ separators

Choose the spots for the 6 cookies (or 3 separators)

Num ways:

$$\binom{6+3}{6} = \binom{9}{6} = \frac{9 \cdot 8 \cdot 7}{3 \cdot 2 \cdot 1} = 84$$

Ex 5: How many nonnegative integer solns does the eqn. $x_1 + x_2 + x_3 = 11$ have

11 "stones" $3-1=2$ "sticks"

* * | * * * | * * * * * *

$$2 + 3 + 6 = 11$$

$$\text{Num ways: } \binom{11+(3-1)}{11} = \binom{13}{11} = 78$$

Permutations of partially indistinguishable objects:

e.g. A A B B B C B A B C B A

For n total objects, k types, n_i of the i th type,

there are $\frac{n!}{n_1! n_2! \dots n_k!} =: \binom{n}{n_1, \dots, n_k}$ permutations (**)

$$\text{(Note: } \binom{n}{k, n-k} = \binom{n}{k} \text{)}$$

Idea: take a permutation:

$n!$ ways

BABCBA

Swapping around the first type of object doesn't change the permutation

$n_1!$ ways to do this

Divide, by division rule

Same for n_2, n_3, \dots

all
the
same

BABCBA
BABCBA
BABCBA
BABCBA
BABCBA
BABCBA

Next, we want to put n objects into k boxes

Distinguishable objects into distinguishable boxes:

k^n ways

If we want n_1, n_2, \dots, n_k elts. in box $1, 2, \dots, k$:

$\binom{n}{n_1, \dots, n_k}$ ways

Indistinguishable objects into distinguishable boxes:

Sticks and stones: $\binom{n+k-1}{n}$

Other two cases are harder: use ad hoc methods
Distinguishable objects and indistinguishable boxes:

Ex 10: How many ways are there to put four
(distinguishable!) students into at most three groups?

Sol'n:

All four in one group: 1 way

ABCD

3 in one group, 1 in another: 4 ways

ABC, D ACD, B

ABD, C BCD, A

2 in one group, 2 in another: 3 ways

AB, CD

AC, BD

AD, BC

2 in one group, 1 in another,

1 in a third: 6 ways

AB, C, D BC, A, D

AC, B, D BD, A, C

AD, B, C CD, A, B

not 6 since

BC, AD is

the same as

AD, BC

Total: $1 + 4 + 3 + 6 = 14$

Indistinguishable objects into indistinguishable boxes:

Ex 11: How many ways are there to pack 6 identical copies of a book into (at most) 4 identical boxes?

Sol'n: List the possibilities:

6, 0, 0, 0 (the same as 0, 6, 0, 0)

5, 1

4, 2

4, 1, 1

3, 3

3, 2, 1

3, 1, 1, 1

2, 2, 2

2, 2, 1, 1

boxes listed in
descending order

"integer partitions"