Problem §6.2: 2: Show that if there are 30 students in a class, then at least two have last names that begin with the same letter.

Problem §6.2: 6: Let d be a positive integer. Show that among any group of $d+1$ (not necessarily consecutive) integers there are two with exactly the same remainder when they are divided by d.

Problem §6.2: 8: Show that if f is a function from S to T, where S and T are finite sets with $|S| > |T|$, then there are elements s_1 and s_2 in S such that $f(s_1) = f(s_2)$, or in other words, f is not one-to-one.

Problem §6.2: 14:

(a) Show that if seven integers are selected from the first 10 positive integers, there must be at least two pairs of these integers with the sum 11.

(b) Is the conclusion in (a) true if six integers are selected rather than seven?

Problem §6.3 - 16: How many subsets with an odd number of elements does a set with 10 elements have?

Problem §6.3 - 20: How many bit strings of length 10 have

(a) exactly three "0"s?

(b) more "0"s than "1"s?

(c) at least seven "1"s?

(d) at least three "1"s?

Problem §6.3 - 24: How many ways are there for ten women and six men to stand in a line so that no two men stand next to each other?

Problem §6.3 - 42: Find a formula for the number of ways to seat r of n people around a circular table, where seatings are considered the same if every person has the same two neighbors without regard to which side these neighbors are sitting on.

Problem §6.4 - 10: Give a formula for the coefficient of x^k in the expansion of $(x+1/x)^{100}$, where k is an integer.

Problem §6.4 - 12: The row of Pascal's triangle containing the binomia coefficients $\binom{10}{k}$, for $0 \leq k \leq 10$, is:

$$1 \quad 10 \quad 45 \quad 120 \quad 210 \quad 252 \quad 210 \quad 120 \quad 45 \quad 10 \quad 1$$

Use Pascal's identity to produce the row immediately following this row in Pascal's triangle.

Problem §6.4 - 22: Prove the identity $\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$, whenever n, r, and k are non-negative integers with $r \leq n$ and $k \leq r$,

(a) using a combinatorial argument.

(b) using an argument based on the formula for the number of r-combinations of a set with n elements.

Problem §6.4 - 27(a): Prove the **hockeystick identity**

$$\sum_{k=0}^{r} \binom{n+k}{k} = \binom{n+r+1}{r}$$

whenever n and r are positive integers, using a combinatorial argument.

Problem §6.5 - 10(a,c,d): A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose

(a) a dozen croissants?

(c) two dozen croissants with at least two of each kind?

(d) two dozen croissants with no more than two broccoli croissants?

Problem §6.5 - 20: How many solutions are there to the inequality

$$x_1 + x_2 + x_3 \leq 11,$$

where x_1, x_2, and x_3 are non-negative integers?

Problem §6.5 - 26: How many positive integers less than $1,000,000$ have exactly one digit equal to 9 and have a sum of digits equal to 13?

Problem §6.5 - 46: A shelf holds 12 books in a row. How many ways are there to choose five books so that no two adjacent books are chosen?