Problem §5.1: 20: Prove that $3^n < n!$ if n is an integer greater than 6.

Problem §5.1: 34: Prove that 6 divides $n^3 - n$ whenever n is a nonnegative integer.

Problem §5.1: 49: What is wrong with this "proof" that all horses are the same color?

Let $P(n)$ be the proposition that all the horses in a set of n horses are the same color.

Basis Step: Clearly, $P(1)$ is true.

Inductive Step: Assume that $P(k)$ is true, so that all the horses in any set of k horses are the same color. Consider any $k+1$ horses: number these horses as $1, 2, 3, \ldots, k, k+1$. Now the first k of these horses all must have the same color. Because the set of the first k horses and the set of the last k horses overlap, all $k+1$ must be the same color. This shows that $P(k+1)$ is true and finishes the proof by induction.

Problem §5.1: 51: What is wrong with this "proof"?

"*Theorem*": For every positive integer n, if x and y are positive integers with $\max(x, y) = n$, then $x = y$.

Basis Step: Suppose that $n = 1$. If $\max(x, y) = 1$ and x and y are positive integers, we have $x = 1$ and $y = 1$.

Inductive Step: Let k be a positive integer. Assume that whenever $\max(x, y) = k$ and x and y are positive integers, then $x = y$. Now let $\max(x, y) = k + 1$, where x and y are positive integers. Then $\max(x - 1, y - 1) = k$, so by the inductive hypothesis $x - 1 = y - 1$. It follows that $x = y$, completing the inductive step.

Problem §5.2: 8: Suppose that a store offers gift certificates in denominations of 25 and 40 dollars. Determine the possible total amounts you can form using these gift certificates. Prove your answer using strong induction.

Problem §5.2: 10: Assume that a chocolate bar consists of n squares arranged in a rectangular pattern. The entire bar, a smaller rectangular piece of the bar, can be broken along on a vertical or horizontal line separating the squares. Assuming that only one piece can be broken at a time, determine how many breaks you must successively make to break the bar into n separate squares. Use strong induction to prove your answer.

Problem §6.1: 8: How many different three-letter initials with none of the letters repeated can people have?

Problem §6.1: 14: How many bit strings of length n, where n is a positive integer, start and end with 1s?

Problem §6.1: 16: How many strings are there of four lowercase letters that have the letter x in them?

Problem §6.1: 26: How many strings of four decimal digits

(a) do not contain the same digit twice?

(b) end with an even digit?

(c) have exactly three digits that are 9s?

Problem §6.1: 30: How many license plates can be made using either three uppercase English letters followed by three digits or four uppercase English letters followed by two digits?

Problem §6.1: 36: How many functions are there from the set $\{1, 2, \ldots, n\}$, where n is a positive integer, to the set $\{0, 1\}$?

Problem §6.1: 37: How many functions are there from the set $\{1, 2, \ldots, n\}$, where n is a positive integer, to the set $\{0, 1\}$

(a) that are one-to-one?

(b) that assign 0 to both 1 and n?

(c) that assign 1 to exactly one of the positive integers less than n?

Problem §6.1: 40: How many subsets of a set with 100 elements have more than one element?

Problem §6.1: 44: How many ways are there to seat four of a group of ten people around a circular table where two seatings are considered the same when everyone has the same immediate left and immediate right neighbor?