Problem §3.1: 2: Determine which characteristics of an algorithm described in the text the following procedures have and which they lack.

(a)
```
procedure double(n: positive integer)
while n > 0
    n := 2n
```

(b)
```
procedure divide(n: positive integer)
while n >= 0
    m : = 1/n
    n := n-1
```

(c)
```
procedure sum(n: positive integer)
sum := 0
while i < 10
    sum := sum + i
```

(d)
```
procedure choose(a,b: integers)
x := either a or b
```

Problem §3.1: 24: Describe an algorithm that determines whether a function from a finite set to another finite set is one-to-one.

Problem §3.1: 52(a,d): Use the greedy algorithm to make change using quarters, dimes, nickels, and pennies for

(a) 87 cents.

(d) 33 cents.

Problem §3.1: 54(a,d): Use the greedy algorithm to make change using quarters, dimes, and pennies (but no nickels) for

(a) 87 cents.

(d) 33 cents.

Problem §3.2: 2(a,b,e,f): Determine whether each of these functions is $O(x^2)$:

(a) $f(x) = 17x + 11$.

(b) $f(x) = x^2 + 1000$.

(e) $f(x) = 2^x$.

(f) $f(x) = \lfloor x \rfloor \cdot \lceil x \rceil$.

Problem §3.2: 8: Find the least integer n such that $f(x)$ is $O(x^n)$ for each of these functions.

(a) $f(x) = 2x^2 + x^3 \log x$.

(b) $f(x) = 3x^5 + (\log x)^4$.

(c) $f(x) = (x^4 + x^2 + 1)/(x^4 + 1)$.

(d) $f(x) = (x^3 + 5\log x)/(x^4 + 1)$.

Problem §3.2: 17: Suppose that $f(x)$, $g(x)$, and $h(x)$ are functions such that $f(x)$ is $O(g(x))$ and $g(x)$ is $O(h(x))$. Show that $f(x)$ is $O(h(x))$.

Problem §3.2: 26: Give a big-O estimate for each of these functions. For the function g in your estimate $f(x)$ is $O(g(x))$, use a simple function g of the smallest order.

(a) $f(x) = (n^3 + n^2 \log n)(\log n + 1) + (17\log n + 19)(n^3 + 2)$.

(b) $f(x) = (2^n + n^2)(n^3 + 3^n)$.

(c) $f(x) = (n^n + n2^n + 5^n)(n! + 5^n)$.

Problem §3.2: 28(a,b,c,d): Determine whether each of the following functions is $\Omega(x)$ and whether it is $\Theta(x)$.

(a) $f(x) = 10$.

(b) $f(x) = 3x + 7$.

(c) $f(x) = x^2 + x + 1$.

(d) $f(x) = 5\log x$.

Problem Extra: Explain what it means for a function to be

(a) $O(1)$.

(b) $\Omega(1)$.

(c) $\Theta(1)$.

Problem §5.1: 4: Let $P(n)$ be the statement that $1^3 + 2^3 + \cdots + n^3 = (n(n+1)/2)^2$ for the positive integer n.

(a) What is the statement $P(1)$?

(b) Show that $P(1)$ is true, completing the basis step of the proof.

(c) What is the inductive hypothesis?

(d) What do you need to prove in the inductive step?

(e) Complete the inductive step, identifying where you use the inductive hypothesis.

(f) Explain why these steps show that this formula is true whenever n is a positive integer.

Problem §5.1: 6: Prove that $1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n! = (n+1)! - 1$ whenever n is a positive integer.

Problem §5.1: 8: Prove that $2 - 2 \cdot 7 + 2 \cdot 7^2 - \cdots + 2(-7)^n = (1 - (-7)^{n+1})/4$ whenever n is a nonnegative integer.