Problem §2.3: 2: Determine whether f is a function from $\mathbb{Z}$ to $\mathbb{R}$ if

(a) $f(n) = \pm n$

(b) $f(n) = \sqrt{n^2 + 1}$

(c) $f(n) = \frac{1}{n^2 - 4}$

Problem §2.3: 12: Determine whether each of these functions from $\mathbb{Z}$ to $\mathbb{Z}$ is one-to-one.

(a) $f(n) = n - 1$.

(b) $f(n) = n^2 + 1$.

(c) $f(n) = n^3$.

(d) $f(n) = \lceil n/2 \rceil$.

Problem §2.3: 14(a,b,c,d): Determine whether $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is onto if

(a) $f(m, n) = 2m - n$.

(b) $f(m, n) = m^2 - n^2$.

(c) $f(m, n) = m + n + 1$.

(d) $f(m, n) = |m| - |n|$.

Problem §2.3: 20: Give an example of a function from $\mathbb{N}$ to $\mathbb{N}$ that is

(a) one-to-one but not onto.

(b) onto but not one-to-one.

(c) both onto and one-to-one (but not the identity function).

(d) neither one-to-one nor onto.

Problem §2.3: 22(a,b): Determine whether each of these functions is a bijection from $\mathbb{R}$ to $\mathbb{R}$.

(a) $f(x) = -3x + 4$.

(b) $f(x) = -3x^2 + 7$.

Problem §2.3: 36: Find $f \circ g$ and $g \circ f$ where $f(x) = x^2 + 1$ and $g(x) = x + 2$ are functions from $\mathbb{R}$ to $\mathbb{R}$.

Problem §2.3: 39: Show that the function $f(x) = ax + b$ from $\mathbb{R}$ to $\mathbb{R}$ is invertible, where a and b are constants, with $a \neq 0$, and find the inverse of f.

Problem §2.3: 40(a): Let f be a function from the set A to the set B. Let S and T be subsets of A. Show that $f(S \cup T) = f(S) \cup f(T)$.

Problem §2.3: 44(b): Let f be a function from A to B. Let S and T be subsets of B. Show that $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$.