MATH 213 – DISCRETE MATH – Fall 2024 – Quiz 9 – Wednesday, Dec. 4

This quiz contains 3 questions – You have 15 minutes

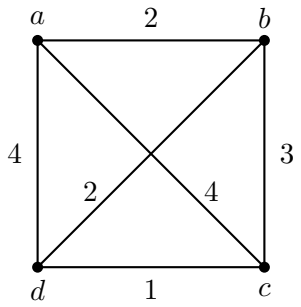
Name: _____

Problem 1.

(a) Give the definition of a planar graph.
Solution: A planar graph is a graph which can be drawn in the plane without any edges crossing.

(b) State Euler's formula for the numbers of vertices v, edges e, and regions r in a connected planar graph.
Solution: For such a graph, we have $v - e + r = 2$.

Problem 2. Find the length of the shortest Hamiltonian circuit in the weighted graph below by checking all possibilities.



Solution: In class, we discussed two simplifications. First, we can assume the circuit starts and ends at a. Second, each circuit has the same length as its reverse, so we can assume b appears before d. With these simplifications, there are three Hamiltonian circuits to check:

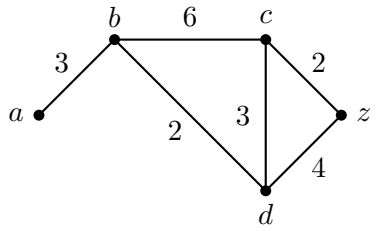
$$a, b, c, d : 2 + 3 + 1 + 4 = 10$$

$$a, b, d, c : 2 + 2 + 1 + 4 = 9$$

$$a, c, b, d : 4 + 3 + 2 + 4 = 13$$

The shortest of these has length 9.

Problem 3. Apply Dijkstra's algorithm to find the length of the shortest path from a to z in the weighted graph below. The work you should show is to give the the set S and the quantities $L(v)$ for all vertices v *after each round of the algorithm*.



Solution: At start: $S = \emptyset,\ L(a) = 0,\ L(b) = \infty,\ L(c) = \infty,\ L(d) = \infty,\ L(z) = \infty$.
After Round 1: $S = \{a\},\ L(a) = 0,\ L(b) = 3,\ L(c) = \infty,\ L(d) = \infty,\ L(z) = \infty$.
After Round 2: $S = \{a, b\},\ L(a) = 0,\ L(b) = 3,\ L(c) = 9,\ L(d) = 5,\ L(z) = \infty$.
After Round 3: $S = \{a, b, d\},\ L(a) = 0,\ L(b) = 3,\ L(c) = 8,\ L(d) = 5,\ L(z) = 9$.
After Round 4: $S = \{a, b, c, d\},\ L(a) = 0,\ L(b) = 3,\ L(c) = 8,\ L(d) = 5,\ L(z) = 9$.
After Round 5: $S = \{a, b, c, d, z\},\ L(a) = 0,\ L(b) = 3,\ L(c) = 8,\ L(d) = 5,\ L(z) = 9$.

Therefore, the distance from a to z is 9.