MATH 213 – DISCRETE MATH – Fall 2024 – Quiz 6 – Friday, Nov. 1

This quiz contains 3 questions – You have 15 minutes

Name: _____

Problem 1. Let R be a relation on a set A. State the following definitions. (No need to define a relation)

(a) R is reflexive
Solution: aRa for all $a \in A$

(b) R is symmetric
Solution: If aRb, then bRa

(c) R is transitive
Solution: If aRb and bRc, then aRc

(d) R is an equivalence relation
Solution: R is reflexive, symmetric, and transitive.

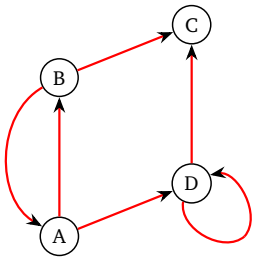
Problem 2. Let $A = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$ be the set of all sequences of three coin flips. Let $\sim$ be the equivalence relation:

$$a \sim b \qquad \text{if and only if} \qquad a \text{ and } b \text{ have the same number of heads.}$$

List all the equivalence classes using roster notation (i.e. list the classes and all their elements explicitly).
Solution: There are 4 equivalence classes, corresponding to a total of 3, 2, 1, and 0 heads, respectively:

$$\{HHH\}, \{HHT, HTH, THH\}, \{HTT, THT, TTH\}, \{TTT\}$$

Problem 3. For the relation given by the following digraph, give the corresponding matrix.



Solution: The corresponding matrix is:

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix},$$

where both rows and columns are ordered: A, B, C, D.