Name:

Problem 1.

(a) State the definition of the conditional probability $P(E|F)$ *(this should be a formula in terms of other probabilities involving E and F)*
    Solution:
    $$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

(b) State Bayes' Theorem *(either of the forms we used in class is fine)*
    Solution:
    $$P(E|F) = \frac{P(F|E)P(E)}{P(F)} = \frac{P(F|E)P(E)}{P(F|E)P(E) + P(F|\overline{E})P(\overline{E})}$$

Problem 2. Let $\{a_n\}$ be the sequence given by the recurrence relation

$$a_n = a_{n-1}a_{n-2}, \qquad \text{and initial conditions} \qquad a_1 = 1, a_2 = 2.$$

Compute $a_3, a_4, a_5,$ and a_6.
Solution:

$$a_3 = a_1 a_2 = 1 \cdot 2, \qquad a_4 = a_2 a_3 = 2 \cdot 2 = 4, \qquad a_5 = a_3 a_4 = 2 \cdot 4 = 8, \qquad a_6 = a_4 a_5 = 4 \cdot 8 = 32$$

(Notice that if we write the terms of this sequence as powers of 2, the exponents form the Fibonacci sequence: $0, 1, 1, 2, 3, 5, \ldots$)

Problem 3. For each of the following recurrence relations, circle whether it is linear homogeneous, linear inhomogeneous (meaning linear but not homogeneous), or neither. *(Circle one option for each part)*

(a) $f_n = f_{n-1} + f_{n-2}$

        (Linear homogeneous)   Linear inhomogeneous   Neither

(b) $H_n = 2H_{n-1} + 1$

        Linear homogeneous   (Linear inhomogeneous)   Neither

(c) $C_{n+1} = C_0 C_n + C_1 C_{n-1} + \cdots + C_n C_0$

        Linear homogeneous   Linear inhomogeneous   (Neither)

(d) $a_n = 6a_{n-1} - 9a_{n-2} + n3^n$

        Linear homogeneous   (Linear inhomogeneous)   Neither