MATH 213 – DISCRETE MATH – Fall 2024 – Quiz 3 – Wednesday, Sept. 18

This quiz contains 3 questions – You have 15 minutes

Name: ____

Problem 1. Define what it means for $f(x)$ to be

a. $O(g(x))$

Solution: f is $O(g)$ if and only if there exist C, k such that if $x > k$, $|f(x)| \leq C|g(x)|$.

b. $\Omega(g(x))$

Solution: f is $\Omega(g)$ if and only if there exist C, k such that if $x > k$, $|f(x)| \geq C|g(x)|$.

c. $\Theta(g(x))$

Solution: $f(x)$ is $\Theta(g(x))$ if and only if f is $O(g)$ and $\Omega(g)$.

Problem 2. What is the difference between (ordinary) *mathematical induction* and *strong induction*?

Solution: In mathematical induction, the inductive hypothesis is that $P(k)$ is true. In mathematical induction, the inductive hypothesis is that $P(1), P(2), \ldots, P(k)$ are all true. Every other aspect is the same.

Problem 3. Consider the following (somewhat poorly-explained) proof by induction of the statement

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

We number the lines of the proof.

Proof.

1. Let $P(n)$ be the statement $1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$.
2. If $n = 1$, then $1^2 = \frac{1 \cdot 2 \cdot 3}{6}$, so $P(1)$ is true.
3. Now let $k \geq 1$, and assume that $P(k)$ is true.
4. Then $1^2 + 2^2 + \cdots + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$.
5. Simplifying the right side of the previous line, we obtain $1^2 + 2^2 + \cdots + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$, so $P(k+1)$ is true.

□

For the proof above, answer the following questions *(no work necessary)*.

a. Which line(s) correspond to the base case?

Solution: Line 2

b. Which line(s) correspond to the inductive step?

Solution: Lines 3-5

c. In which line(s) is the inductive hypothesis USED?

Solution: Line 4