

MATH 213 – DISCRETE MATH – Fall 2024 – Quiz 2 – Wednesday, Sept. 11
This quiz contains 3 questions – You have 15 minutes

Name: _____

Problem 1. Let $f : A \rightarrow B$. State the following definitions.

a. f is injective

Solution: For every $a_1, a_2 \in A$, if $f(a_1) = f(a_2)$, then $a_1 = a_2$ (or equivalent statement)

b. f is surjective

Solution: For every $b \in B$, there exists $a \in A$ such that $f(a) = b$ OR $f(A) = B$ OR the range of f is B (or equivalent statement)

c. f is bijective

Solution: f is injective and surjective (or equivalent statement)

Problem 2. For each of the following functions $f : A \rightarrow B$, is f injective, surjective, both, or neither? (no work needed)

a. $A = \{1, 2, 3\}, B = \{1, 2\}; f(1) = 1, f(2) = 2, f(3) = 2$

Solution: Surjective, but not injective

b. $A = \{1, 2, 3\}, B = \{1, 2, 3\}; f(1) = 1, f(2) = 2, f(3) = 2$

Solution: Neither

c. $A = B = \mathbb{N}, f(x) = x^2 + 1$

Solution: Injective, but not surjective

Problem 3. Recall the binary-search algorithm:

Input: an integer x ; a list a_1, \dots, a_n of distinct integers with $a_1 < a_2 < \dots < a_n$.

Output: the location i of x in the list (or 0 if it's not in the list).

Algorithm:

Let $i := 1$

Let $j := n$

While $i < j$

 Let $m := \lfloor \frac{i+j}{2} \rfloor$

 If $x > a_m$, set $i := m + 1$

 Otherwise, set $j := m$

If $x = a_i$, location := i

Otherwise, location := 0

return location

Implement this algorithm with $x = 8$ and the list $0, 1, 2, 4, 5, 6, 8, 9$ ($n = 8$). Show the result of each while loop.

Solution:

Start: $i = 1, j = 8$

Round 1: $i = 1, j = 8, m = 4, a_m = 4$. Since $x = 8 > 4$, we set $i := m + 1 = 5$

Round 2: $i = 5, j = 8, m = 6, a_m = 6$. Since $x = 8 > 6$, we set $i := m + 1 = 7$

Round 3: $i = 7, j = 8, m = 7, a_m = 8$. Since $x = 8 \leq 8$, we set $j := m = 7$

Now, we have $i = j = 7$, so we exit the while loop. Since $x = 8 = a_7$, we return 7.