

Note: the distribution of these problems may not match the distribution of exam topics.

Problem §2.1: Let $A = \{2, 6\}$ and $B = \{3, 1\}$.

- Find $\mathcal{P}(A)$ and $|\mathcal{P}(A)|$.
- Find $A \times B$.
- Is $A \times B = B \times A$? Why or why not?

Problem §2.1: Let $C = \{n \in \mathbb{N} : n < 6\}$ and $D = \{1, 3, 5\}$.

- Write C in set roster notation.
- Is $D \subseteq C$? Why or why not?
- Draw a Venn diagram representing sets C and D . (*Hint: This diagram should reflect the relationship that you determined in part (b).*)

Problem §2.2: Prove that $A \cup (A \cap B) = A$ by showing that each set is a subset of the other.

Problem §2.3: Consider the function $f : \mathbb{Z} \rightarrow \mathbb{Z}_{\geq 0}$ defined by $z \mapsto |z| + 1$.

- Is f one-to-one? Why or why not?
- Is f onto? Why or why not?

Problem §3.1: 61: Write the deferred acceptance algorithm in pseudocode.

Problem §3.2: Give a big- O estimate for $f(x) = (7n^n + n2^n + 3^n)(n! + 3^n)$. For the function $g(x)$ in your estimate $O(g(x))$, use a simple function g of the smallest order.

Problem §3.2: Use the definition of “ $f(x)$ is $O(g(x))$ ” to show that $f(x) = x^4 + 7x^3 + 8$ is $O(x^4)$.

Problem §5.1: 10:

- Find a formula for

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n(n+1)}$$

by examining the values of this expression for small values of n .

- Prove the formula you conjectured in part (a).

Problem §5.1: 18: Let $P(n)$ be the statement that $n! < n^n$, where n is an integer greater than 1.

- What is the statement $P(2)$?
- Show that $P(2)$ is true, completing the basis step of the proof.
- What is the inductive hypothesis?
- What do you need to prove in the inductive step?

- (e) Complete the inductive step.
- (f) Explain why these steps show that this inequality is true whenever n is an integer greater than 1.

Problem §5.1: 22: For which nonnegative integers is $n^2 \leq n!$? Prove your answer.

Problem §5.1: 36: Prove that 21 divides $4^{n+1} + 5^{2n-1}$ whenever n is a positive integer.

Problem §5.1: 38: Prove that if A_1, A_2, \dots, A_n and B_1, B_2, \dots, B_n are sets such that $A_j \subseteq B_j$ for $j = 1, 2, \dots, n$, then

$$\bigcup_{j=1}^n A_j \subseteq \bigcup_{j=1}^n B_j.$$

Problem §5.1: 57: Use mathematical induction to prove that the derivative of $f(x) = x^n$ equals nx^{n-1} whenever n is a positive integer.

Problem §5.2: 31: Show that strong induction is a valid method of proof by showing that it follows from the well-ordering property.

Problem §6.1: 28: How many license plates can be made using either three digits followed by three uppercase English letters or three uppercase English letters followed by three digits?

Problem §6.2: 31: Show that there are at least six people in California (population: 37 million) with the same three initials who were born on the same day of the year (but not necessarily the same year). Assume that everyone has three initials.

Problem §6.1: 37: How many functions are there from the set $\{1, 2, \dots, n\}$, where n is a positive integer, to the set $\{0, 1\}$

- (a) That are one-to-one?
- (b) That assign 0 to both 1 and n ?
- (c) That assign 1 to exactly one of the positive integers less than n ?

Problem §6.2: 40: Prove that at a party where there are at least two people, there are two people who know the same number of other people there.