Announcement

HW2 due Sunday @ 11:59pm via Gradescope

$f : A \to B$

function domain codomain

Def: $f : A \to B$

f is one-to-one/injective if whenever $a \neq b$, $f(a) \neq f(b)$

f is onto/surjective if $f(A) = B$

$f(A) = \{b \in B \mid f(a) = b \text{ for some } a \in A\}$

codomain = range

f is bijective if it is injective and surjective

Ex: $f : A \to B$ $A = \{a, b, c\}$ $B = \{x, y, z\}$

$f(a) = x$, $f(b) = z$, $f(c) = x$

f is not injective since $f(a) = f(c)$ but $a \neq c$

f is not surjective since $y \notin f(A)$

Ex: $g: \mathbb{R} \to \mathbb{R}$

$g(x) = x + 1$

g is injective since if $g(x) = g(y)$, then $x+1 = y+1$, so $x = y$.

g is surjective since if $z \in \mathbb{R}$, then $g(z-1) = (z-1)+1 = z$

Surjective means range = codomain $f: A \to B$

Always have range $\subseteq$ codomain

WTS: every elt. of codomain is in the range

i.e. for every $b \in B$, $b \in f(A)$ i.e $b = f(a)$ for some $a \in A$

Thus, g is bijective

The book has more information about injectivity for <u>increasing</u>/ <u>decreasing</u> functions

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Bijections have inverse functions

$f: A \to B$ bijection

$f^{-1}: B \to A$ (also a bijection)

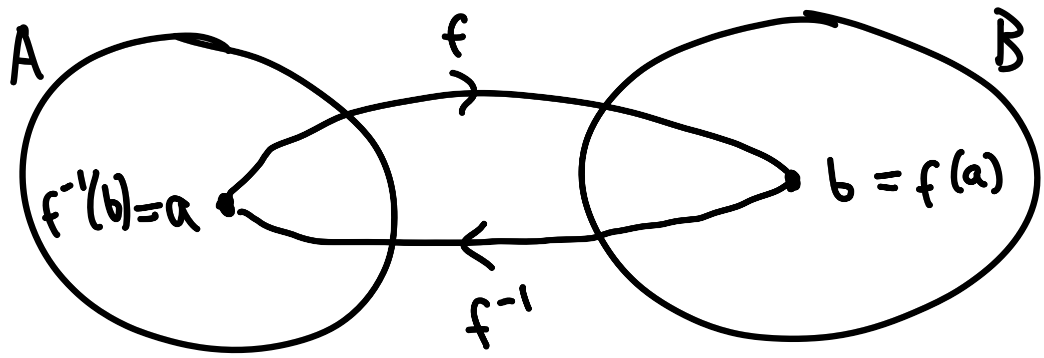
$\{a \in A \mid f(a) \in C\}$

$\overset{\{a \in A \mid f(a) \in C\}}{\shortparallel}$

$\left.\begin{matrix} f^{-1}(C) \\ \text{vs.} \\ f^{-1}: B \to A \end{matrix}\right\}$ not quite the same

f^{-1} "undoes f": if $f(a)=b$, then $f^{-1}(b)=a$

We call a function with an inverse <u>invertible</u>



Ex: set of pos. real nums.

a) $f: \mathbb{R}_+ \to \mathbb{R}_+$ $\quad f(x) = x^2$

is invertible w/ $f^{-1}(x) = \sqrt{x}$ ← pos. sqrt.

b) $A = \{a, b, c\}$ $\quad f: A \to A$

$f(a)=b,\ f(b)=c,\ f(c)=a$

is invertible w/

$f^{-1}(b)=a,\ f^{-1}(c)=b,\ f^{-1}(a)=c$

Composition: apply functions in sequence

Let $f: A \to B$, $\quad g: B \to C$

need to be the same

Then $g \circ f : A \to C$ is given by

$$g \circ f(a) = g(f(a))$$

Ex: $f: \overset{A}{\mathbb{Z}} \to \overset{B}{\mathbb{Z}}$ $\qquad g: \overset{B}{\mathbb{Z}} \to \overset{C}{\mathbb{N}}$

$f(x) = x+1 \qquad g(x) = x^2$

$g \circ f : \mathbb{Z} \to \mathbb{N}$

$(g \circ f)(x) = (x+1)^2$

$f \circ g$ is not defined since domain$(f) \neq$ codomain(g)

§3.1 Algorithms

Def: An <u>algorithm</u> is a finite sequence of precise steps

<u>Properties:</u>

- Input
- Output
- Definiteness: Steps are precisely-defined
- Correctness: Always gives the right answer
- Finiteness: Finite # steps for any input
- Effectiveness: You can actually do each step
- Generality: Works for all possible inputs

Ex: Making change

Idea: We have a value of n "cents" and we want to make change using coins of values $c_1, c_2, c_3, \ldots, c_r$

Greedy Change-Making Algorithm:

procedure change($c_1, c_2, \ldots, c_r$: values of coins, where $c_1 > c_2 > \cdots > c_r$; n: pos. int.)

pos. int.

$a := 5$ means set $a = 5$

for $i := 1$ to r

  $d_i := 0$ (d_i is the num. coins of value c_i)

  while $n \geq c_i$

    $d_i := d_i + 1$ (adds a coin of value i)

    $n := n - c_i$ (c_i less value remaining)

return $d_1, d_2, \ldots, d_r$

This is an example of an optimization problem

Optimization problem: maximize/minimize some parameter

e.g. Give change using the fewest num. of coins possible

Greedy algorithm: Try to solve the optimization problem by making the "best" choice at each step

doesn't always give the optimal solution