Announcements

Final exam: Thursday 12/19 1:30-4:30pm
4025 Campus Instructional Facility

Wed. class will be review
Policies/practice problems to come later

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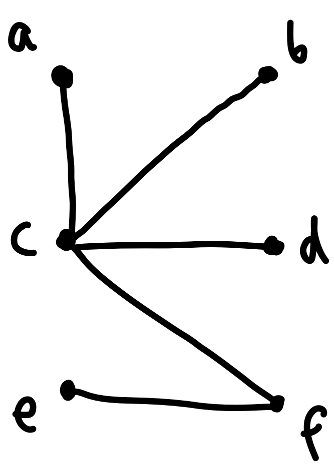
§11.1: Trees

Def: A <u>tree</u> is a conn. (undir.) graph w/ no simple circuits

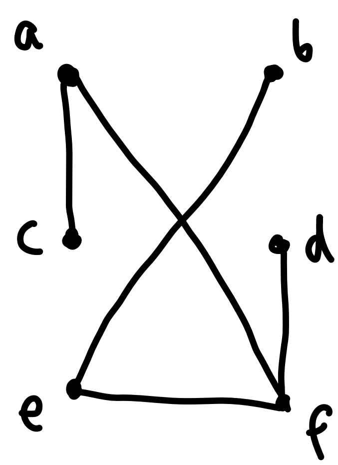
A <u>forest</u> is a graph consisting of one or more trees

Class activity: Tree or no tree?

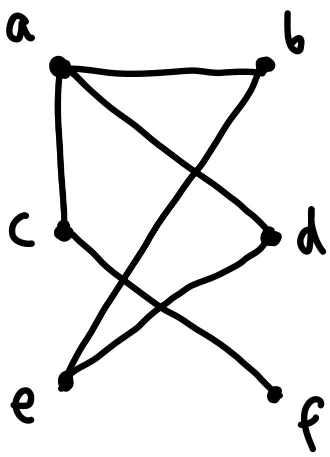
a)



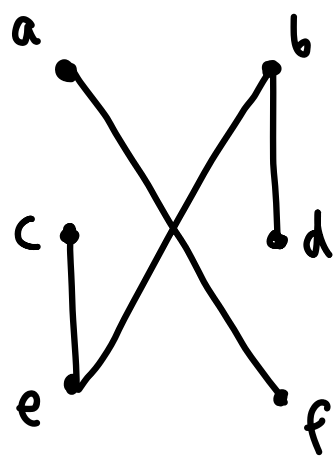
b)



c)



d)



Properties: Suppose T is a tree.

a) T has no loops or mult. edges

b) There is exactly one simple path btwn. any two verts.

c) $|E| = |V| - 1$

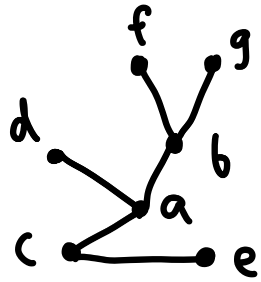
d) Every edge is a cut edge

e) Adding any edge creates a simple circuit

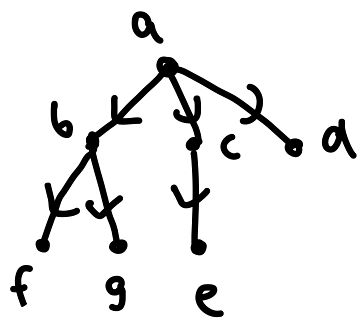
Def: A rooted tree is a tree in which one vertex has been designated the root

Note: We sometimes think of a rooted tree as a digraph where every edge is directed away from the root

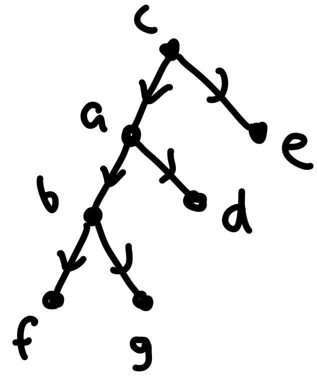
Ex:



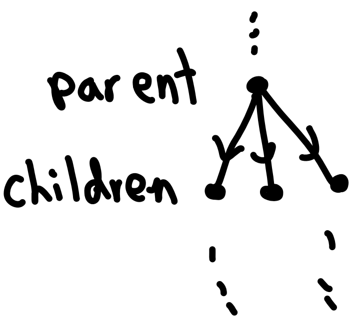
tree

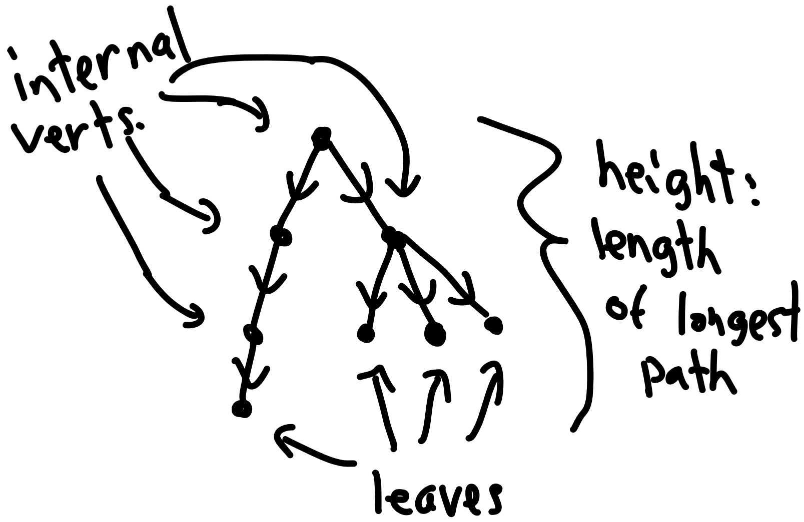


rooted tree
w/ root a



rooted tree
w/ root c



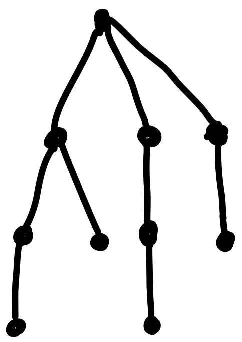


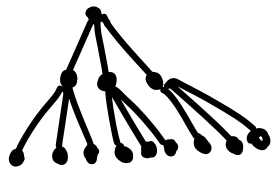
Def: A rooted tree is called an <u>m-ary tree</u> if every internal vertex has $\leq m$ children.
If every internal vertex has $= m$ children, it is called <u>full</u>

An m-ary tree w/ $m = 2$ is a <u>binary tree</u>

An m-ary tree of height h has $\leq m^h$ leaves

Class activity: Count the number of leaves, internal vertices, (total) vertices in

a) 

b) A full 3-ary tree of height 2 

b) A full binary tree of height 3

d) A full binary tree of height 7

Facts:

a) A full binary tree of height h has $2^{h+1} - 1$ vertices, 2^h of which are leaves, and $2^h - 1$ of which are internal vertices.

b) A full m-ary tree of height h has $1 + m + m^2 + \cdots + m^h$ verts. m^h of which are leaves, and $1 + m + \cdots + m^{h-1}$ of which are internal vertices.

c) A full m-ary tree w/ i int. verts. has $mi+1$ verts. and $(m-1)i+1$ leaves

d) A full m-ary tree w/ n verts. has $\frac{n-1}{m}$ int. verts. and $\frac{(m-1)n+1}{m}$ leaves