Quiz today!

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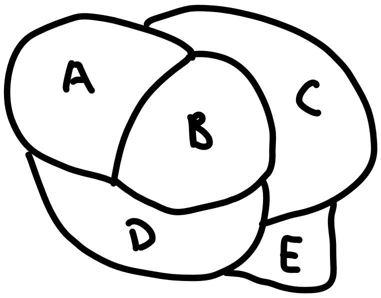
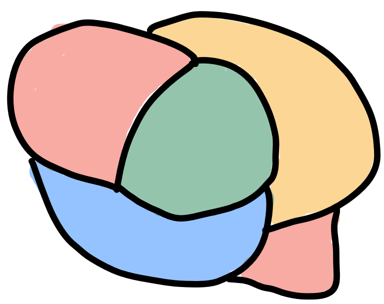
§10.8: Graph coloring

Map: Separation of (part of) the plane into contiguous regions



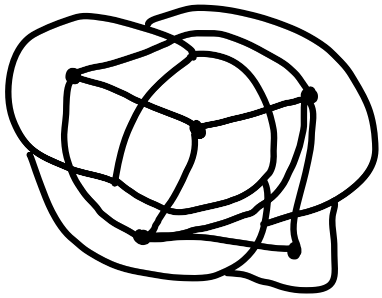
Map coloring: assign each region a color s.t. all adjacent regions have different colors

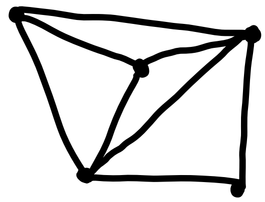


Question: what is the smallest number of colors we need for a given map?

This is secretly a graph theory problem

Def: For a map M, the <u>dual graph</u> of M is the graph formed by putting a vertex in the middle of each region of M, and connecting vertices for adjacent regions





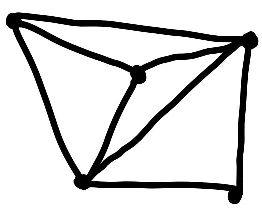
Graph coloring: assign each vertex a color s.t. all adjacent vertices have different colors

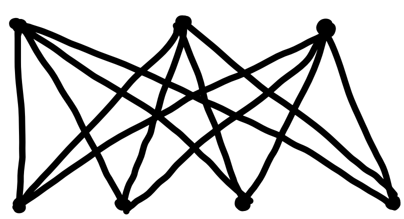
(coloring the dual graph of a map is equiv. to coloring the map itself)

Def: The <u>chromatic number</u> $\chi(G)$ "chi" is the smallest number of colors needed to color G.

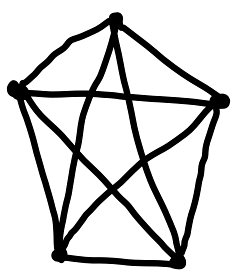
We can do this for any graph; the planar ones are the graphs corresponding to maps.

Class activity: Find $\chi(G)$

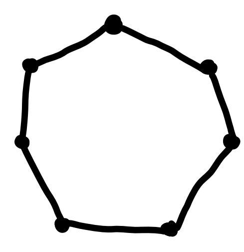




$K_{3,4}$



K_5



C_7

Four-color theorem: For any simple planar graph G, $\chi(G) \leq 4$

1852: Conjectured by Guthrie

1879: ~~Proof given by Kempe~~

1890: Heawood showed that Kempe's proof was flawed (!)
and also proved the five-color thm.

⋮

many years passed

⋮

1976: Appel & Haken proved the theorem!
@UIUC

Proof technique: break up into 1,834 cases,
and check them all by computer

To this day, no non-computer proof exists!