Announcements

Quiz 9 Wednesday

HW11 posted soon (due Sun.)

Midterm 3 grades released + sol'ns posted

Mean: 69.4 / 78
Median: 74 / 78
Std. dev.: 8.2

Q1: 92%
Q2: 85%
Q3: 76%
Q4: 93%
Q5: 93%

Gradelines:

$\left.\begin{array}{l} \text{A/A-}: 70-78 \\ \text{B+/B/B-}: 61-69 \\ \text{C+/C/C-}: 48-60 \\ \text{D+/D/D-}: 30-47 \end{array}\right\}$ out of 78

Gradeline calculator updated

Regrade requests open for 1 week

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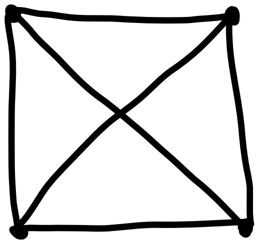
§10.7: Planar graphs

Def: A graph is called <u>planar</u> if it can be drawn w/out edges crossing

Such a drawing is called a <u>planar representation</u>

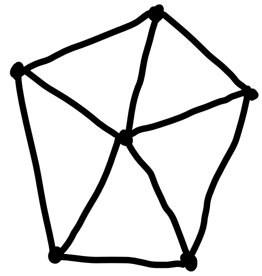
Class activity: are the following graphs planar?

a)



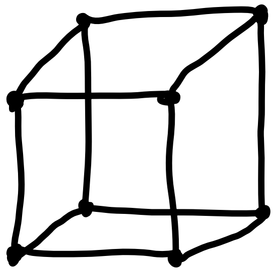
K_4

b)



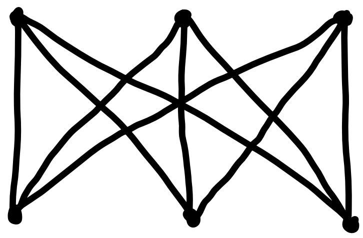
W_5

c)



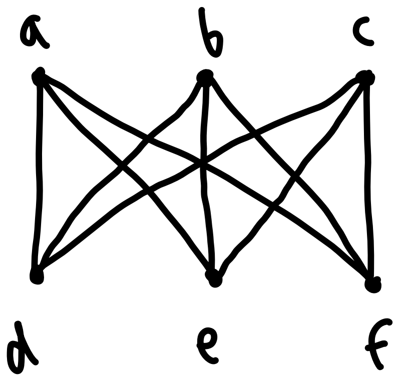
Q_3

d)

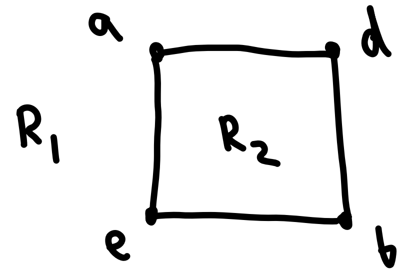


$K_{3,3}$

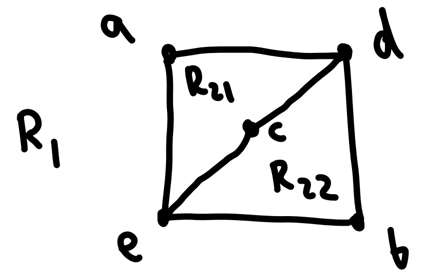
Ex 3: $K_{3,3}$ is nonplanar



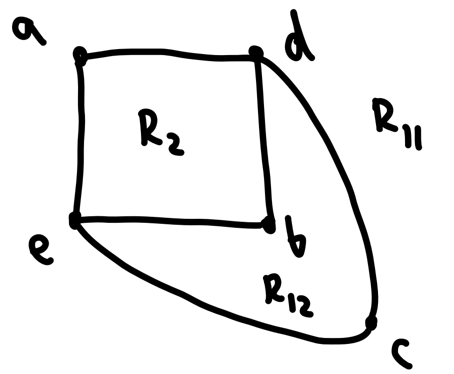
Pf: Suppose $K_{3,3}$ has a planar representation. Then the induced subgraph formed by a, b, d, e is C_4:



Two possible ways to add in c:



or



Finally, we need to place f and connect it to $a, b,$ and c. To do so w/out edge crossings, f must lie in a region w/ a, b, c on the boundary.

For the left diagram:

Region R_1: a, b on bdy, c not

Region R_{21}: a, c on bdy, b not

Region R_{22}: b, c on bdy, a not

For the right diagram:

Region R_2: a, b on bdy, c not

Region R_{11}: a, c on bdy, b not
Region R_{12}: b, c on bdy, a not

Therefore, no such region exists, so $K_{3,3}$ is not planar. $\square$

HW: similar argument for K_5

Now count vertices (v), edges (e), and regions (r) in the planar representations above

planar rep'n of a

Euler's formula: For every connected planar graph,

$$v - e + r = 2$$

Ex 4: Suppose that a conn. planar graph has 20 vertices, each of deg. 3. How many regions does a planar representation have?

Sol'n:

$$v = 20$$

$$e = \frac{1}{2}\sum_{v\in V} \deg(v) = \frac{1}{2}\cdot 20\cdot 3 = 30$$

Apply Euler's formula:

$$20 - 30 + r = 2 \rightsquigarrow r = 12$$

Notice that if the number of vertices is fixed, more edges means more regions

Cor 1: If G is a $\underbrace{\text{connected planar simple}}_{\text{notice all the adjectives!}}$ graph and $v \geq 3$, then $e \leq 3v-6$

Pf: Let the <u>degree</u> be the number of edges on the bdy of the region (counted twice if both "sides" of the edge are in the same region)

Since the graph is simple, if $e \geq 2$, each region has degree ≥ 3, and since each edge contributes a total of 2 to the degree of regions,

$$2e = \sum_{\substack{\text{regions} \\ R}} \deg(R) \geq 3r, \quad \text{so } r \leq \tfrac{2}{3}e$$

By Euler's formula,

$$2 = v - e + r \leq v - e + \tfrac{2}{3}e = v - \tfrac{1}{3}e,$$

so $e \leq 3v-6$. $\square$

Ex 5: Let $G = K_5$.

$v = 5$

$e = \binom{5}{2} = \frac{5!}{2!\,3!} = 10$

$3v-6 = 9 < 10$, so K_5 is not planar

Cor 2 (if time): If G is a conn. simple planar graph, then G has a vertex of $\deg \leq 5$.

Pf: If $v=1$ or 2, this is clearly true. Suppose that $v \geq 3$ and $\deg(u) \geq 6$ for all $u \in V$. Then, by the Handshake Theorem:

$$2e = \sum_{u \in V} \deg(u) \geq \sum_{u \in V} 6 = 6v$$

so $e \geq 3v > 3v-6$, which is impossible. $\square$