Announcements

Midterm 3: Wed. in class

- Covers through Section 10.5
- Reference sheet allowed (one A4 sheet w/ writing on both sides)
- See policy email (practice problems etc.)

Monday: review

Thursday problem session next week will be Tuesday & review

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§10.6: Shortest-path problems

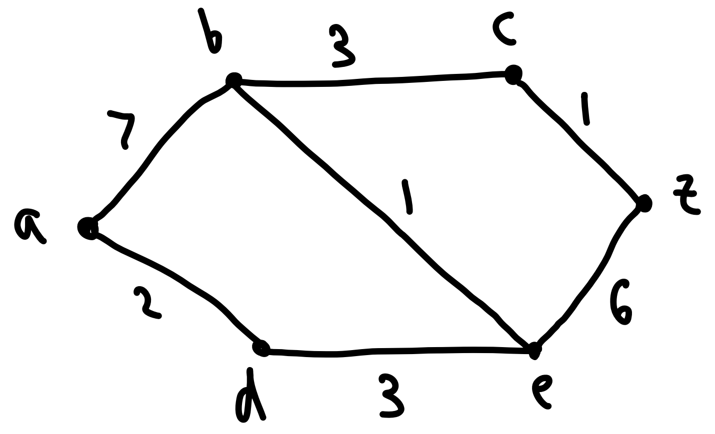
For this section, $G=(V,E)$ is a simple conn. (undir.) graph

Each edge $e \in E$ has a __weight__ $wt(e)$ (always a pos. num.)

Question: Given two vertices, what is the __shortest path__ from one to the other?

(shortest path ← add up the wt of each edge)

Ex:



Shortest path from a to z?

Some possibilities:

$a, b, c, z: 7+3+1 = 11$

$a, d, e, z: 2+3+6 = 11$

$a, b, e, z: 7+1+6 = 14$

$a, d, e, b, c, z: 2+3+1+3+1 = 10$ ✓ shortest path

Note also that the shortest path from a to b is a, d, e, b

Dijkstra's algorithm

Input: weighted conn. simple graph $G=(V,E)$ (all wts. pos.)

start vertex $a \in G$

$w(u,v)$ = length of the edge btwn. u and v (∞ if no such edge)

Output: shortest path distance from a to all other vertices

Algorithm:

$L(a) := 0$

$L(v) := \infty$ for all other vertices v

$L(v_i)$: distance from a to v_i

S: vertices considered so far

$S = \emptyset$

while $S \neq V$

$u :=$ a vertex not in S with $L(u)$ minimal

Add u to S

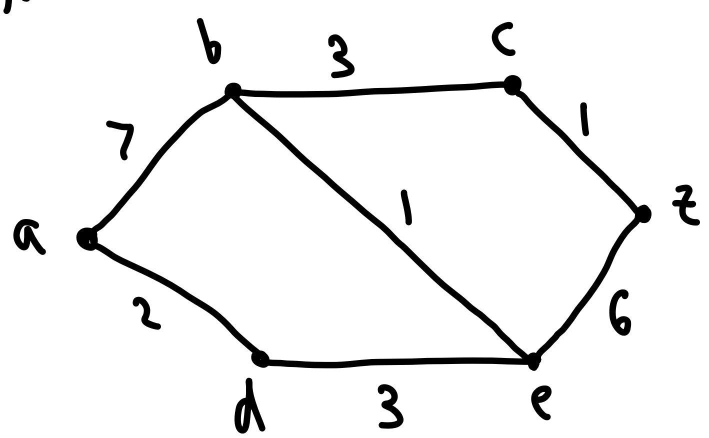
for all vertices v not in S

if $L(u) + w(u,v) < L(v)$, — i.e. passing through u gives a shorter path to v.

then $L(v) := L(u) + w(u,v)$

return $\{(v, L(v)) \mid v \in V\}$ — distances from a to v for all vertices v

Ex (cont.):



$L(a) = 0$ $\quad L(d) = \infty$

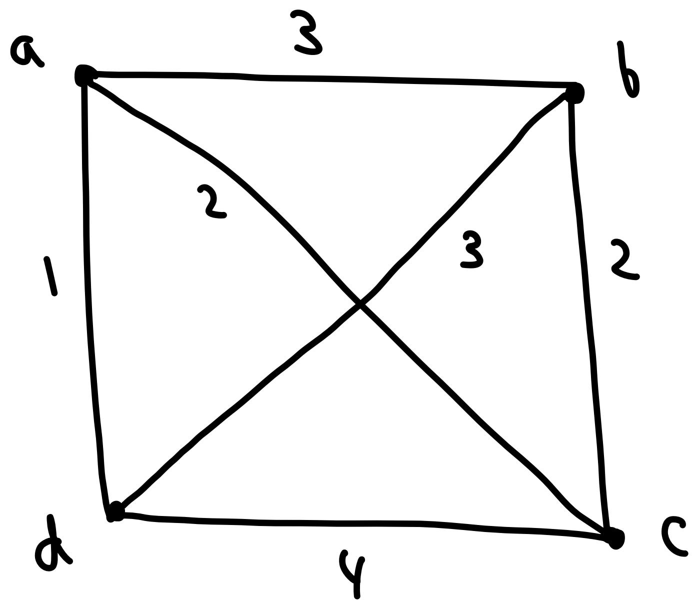
$L(b) = \infty$ $\quad L(e) = \infty$ $\quad\quad S = \emptyset$

$L(c) = \infty$ $\quad L(z) = \infty$

Travelling salesperson problem: Let $G = K_n$ (weighted graph)

Question: What is the shortest Hamiltonian circuit of G?

Ex:



Starting w/ a, there are 6 Hamiltonian circuits

a, b, c, d, a : $3 + 2 + 4 + 1 = 10$

a, b, d, c, a

a, c, b, d, a

a, c, d, b, a

a, d, b, c, a

a, d, c, b, a

Class activity: finish this example