Announcements

Quiz today!

HW10 posted

Midterm 3 Wed. 11/20 in class

Recall: A <u>path</u> is an alternating sequence

$$v_0, e_1, v_1, e_2, \dots, e_n, v_n$$

$v_i \in V$, $e_i \in E$ e_i has endpoints v_{i-1} & v_i

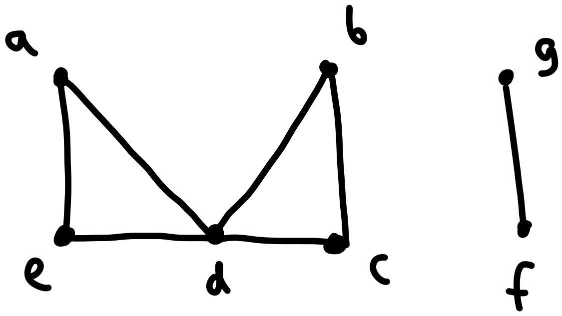
If $v_0 = v_n$, it is a <u>circuit</u>

Def: An Eulerian <u>path/circuit</u> uses each edge exactly once

A Hamiltonian <u>path/circuit</u> uses each vertex exactly once

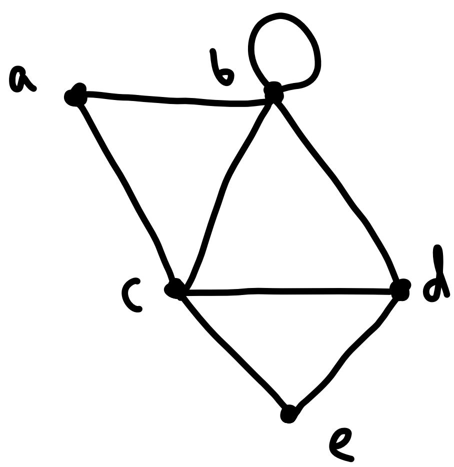
(except, if circuit, $v_0 = v_n$)

Ex: (i)

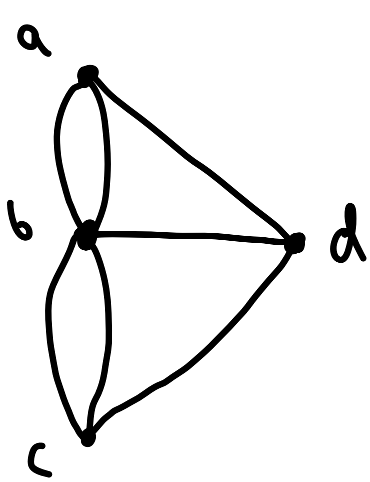


Let's only deal w/ conn. graphs

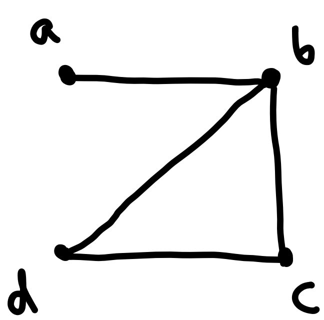
(ii)



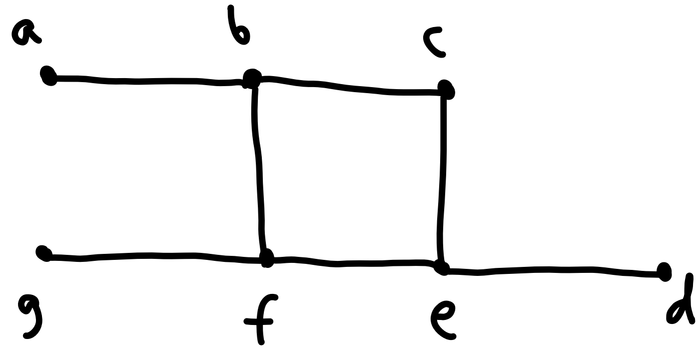
(iii)



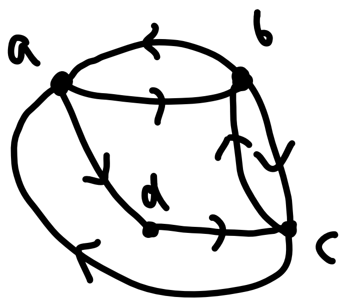
(iv)



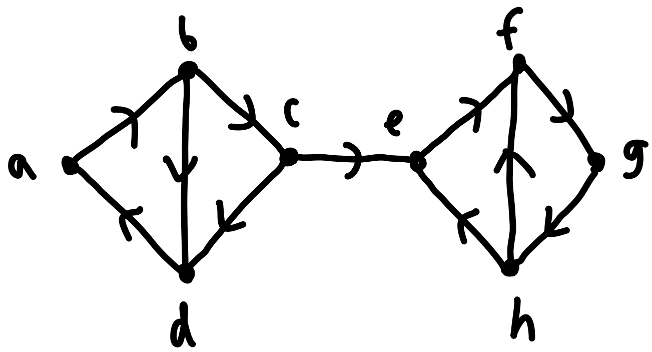
v)



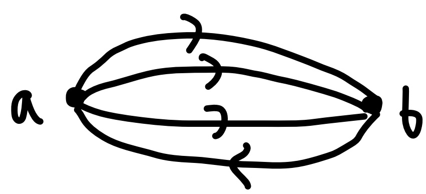
vi)



vii)



viii)



Theorem:

a) A conn. graph has an Eulerian circuit if and only if all degrees are even

b) A conn. graph has an Eulerian path if and only if ≤ 2 degrees are odd

c) A weakly conn. digraph has an Eulerian circuit if and only if $\deg^-(v) = \deg^+(v)$ for all vertices v

d) A weakly conn. digraph has an Eulerian path if and only if $\deg^-(v) = \deg^+(v)$ for all vertices v except for at most two, one of which has $\deg^-(v) = \deg^+(v) + 1$, and the other of which has $\deg^-(v) = \deg^+(v) - 1$

Which graphs have Hamiltonian paths/circuits?

Many classes of graphs do e.g. K_n, C_n, W_n, Q_n

But in general the problem is very hard (NP-complete!)