Announcements

H/W 10 will be posted soon

Quiz this Wed.

Midterm 3 Wed. 11/20 in class

Recall:

Def: Let $G = (V_1, E_1)$ and $H = (V_2, E_2)$ be simple graphs. A function $f: V_1 \to V_2$ is an <u>isomorphism</u> if

a) f is a bijection

b) $f(a)$ and $f(b)$ are adj. if and only if a and b are adj.

If any isomorphism exists, G and H are <u>isomorphic</u>

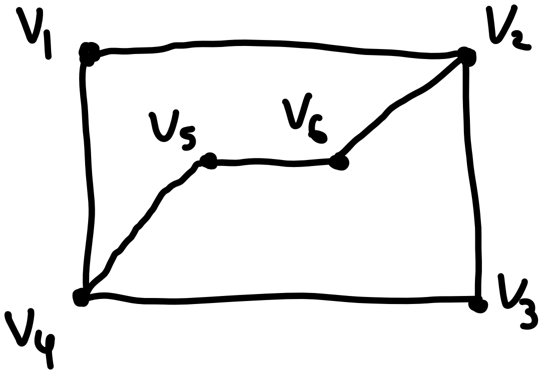
Two ways to show two graphs <u>are</u> isom.

1) Find an isomorphism (examples last time)

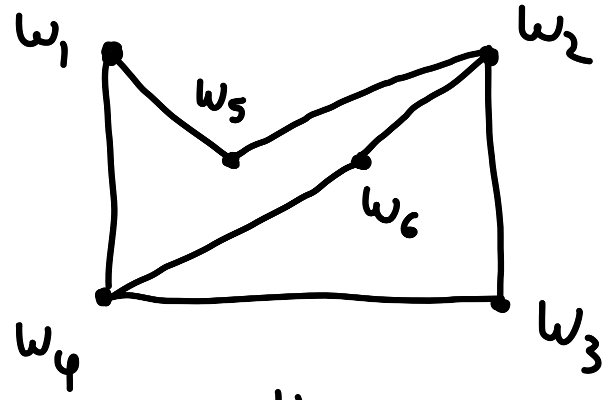
2) Show that the adjacency matrices are the same for some ordering of the vertices

(always same ordering on rows & cols!)

Ex 11:



G



H

$$Adj_G = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$Adj_H = \begin{array}{c} \\ w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{array} \begin{array}{c} \begin{array}{cccccc} w_1 & w_2 & w_3 & w_4 & w_5 & w_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Not the same

But... put the vertices in a different order, and

$$
\begin{array}{c c}
 & \begin{array}{cccccc} w_6 & w_2 & w_3 & w_4 & w_1 & w_5 \end{array} \\
\begin{array}{c} w_6 \\ w_2 \\ w_3 \\ w_4 \\ w_1 \\ w_5 \end{array} &
\begin{bmatrix}
0 & 1 & 0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & 0
\end{bmatrix}
\end{array}
$$

So G and H are isomorphic.

§10.4: Connectivity

Def: A path is an alternating sequence

$$v_0, e_1, v_1, e_2, \ldots, e_n, v_n$$

$v_i \in V$, $e_i \in E$ $\quad$ e_i has endpoints v_{i-1} & v_i

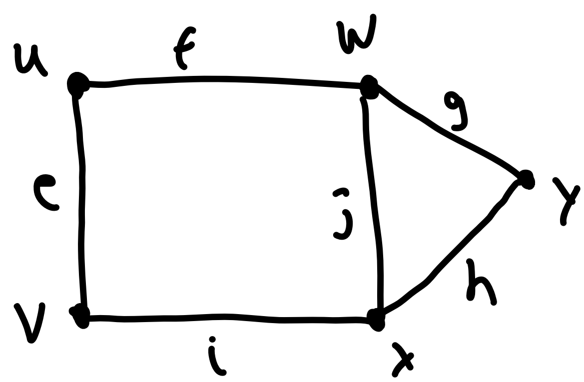
If no repeated edges, the path is simple

If $v_0 = v_n$, it is a <u>circuit</u>

If G is a simple graph, don't need to write the edges, since they're implies

* Note: terminology matches Rosen, different from other sources

Class activity:



Which of the following are paths? Circuits? Simple paths/circuits?

a) u, f, w, j, x, i, v, e, u

b) u, w, x, v, u

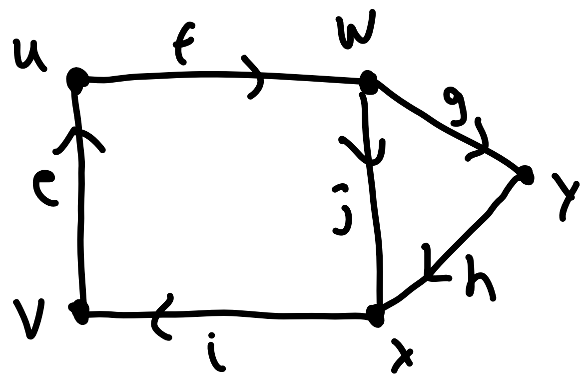
c) u, e, w, i, x, j, v, f, u

d) u, w, x, y, w, x, y

e) u, f, w, f, u, f, w

Def: Paths/circuits are the same in digraphs except they must <u>follow the arrow</u>

Class activity:



Which of the following are paths? Circuits? Simple paths/circuits?

a) u, f, w, j, x, i, v, e, u

b) u, w, x, v, u

c) u, e, w, i, x, j, v, f, u

d) u, w, x, y, w, x, y

e) u, f, w, f, u, f, w

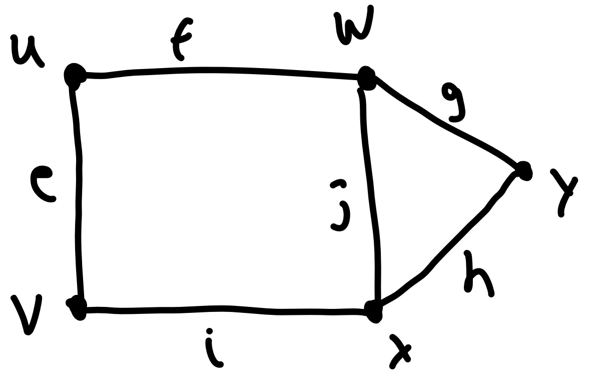
Def: a) A graph is <u>connected</u> if there is a path from every vertex to every other vertex

b) A digraph is <u>strongly connected</u> if there is a path from every vertex to every other vertex

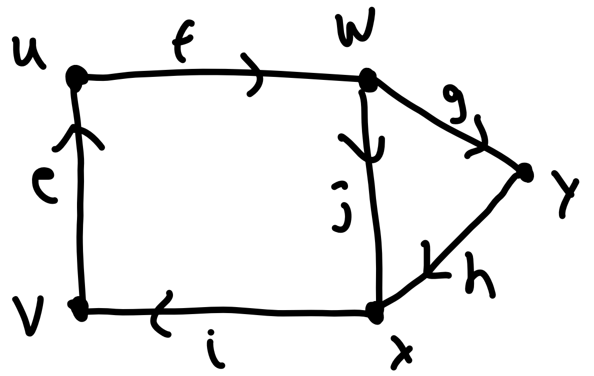
c) A digraph is <u>weakly connected</u> if there is a path from every vertex to every other vertex in the underlying graph (erase the arrows)

d) A <u>cut edge</u>/<u>cut vertex</u> in a graph is an edge/vertex which, when deleted, disconnects the graph

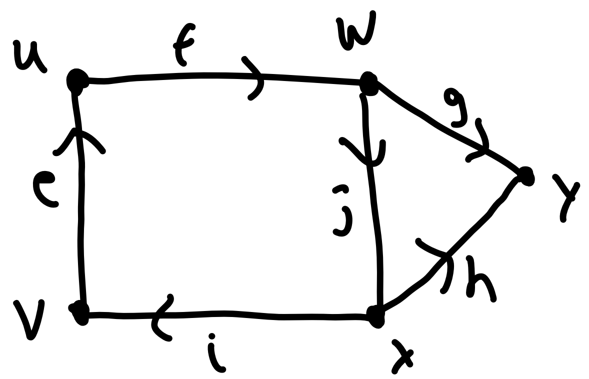
Examples:



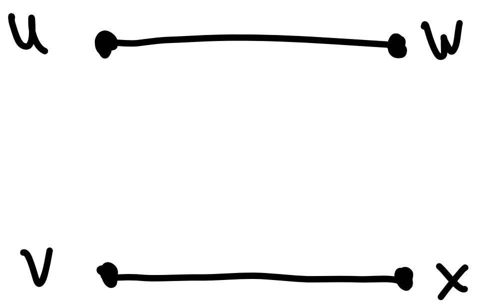
connected, no cut-edges/vertices



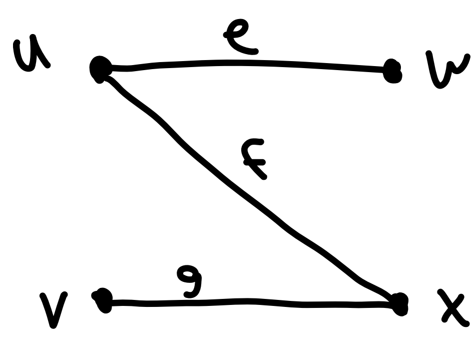
strongly conn.



weakly conn.
since no path from
y to e.g. v



disconnected



connected
cut-vertices: u, x
cut-edges: e, f, g