Announcements

Quiz today!

Midterm 3 Wed. 11/20 in class

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§ 10.3: Representing graphs & graph isomorphism

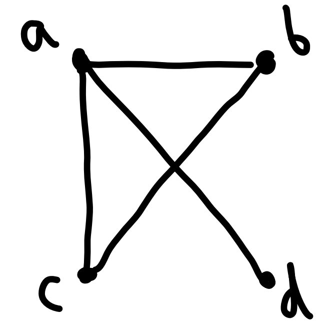
Def: Let G be a graph w/ vertices $v_1, \ldots, v_n$.

The <u>adjacency matrix</u> of G is the matrix $Adj_G = [a_{ij}]$

where a_{ij} = # edges with endpoints v_i & v_j

Ex 3:

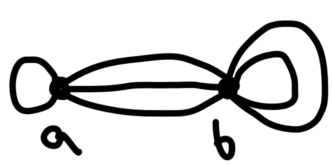
G:



$$Adj_G = \begin{array}{c} \\ a \\ b \\ c \\ d \\ \\ \end{array} \begin{array}{c} \left[\begin{array}{cccc} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right] \\ \begin{array}{cccc} a & b & c & d \end{array} \end{array}$$

Ex:

G:



$$Adj_G = \begin{array}{c} \\ a \\ b \\ \\ \end{array} \begin{array}{c} \left[\begin{array}{cc} 1 & 3 \\ 3 & 2 \end{array}\right] \\ \begin{array}{cc} a & b \end{array} \end{array}$$

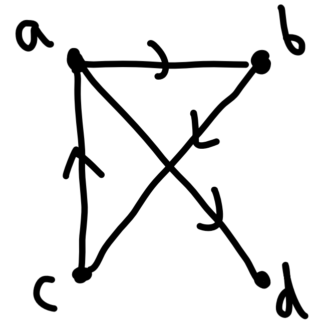
Def: Let D be a digraph w/ vertices $v_1, \dots, v_n$.

The <u>adjacency matrix</u> of D is the matrix $Adj_D = [a_{ij}]$

where a_{ij} = # edges from v_i to v_j

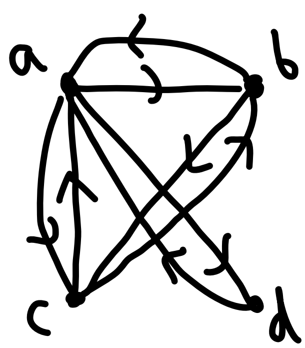
Ex:

D :



$$Adj_D = \begin{array}{cc} \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ & \begin{matrix} a & b & c & d \end{matrix} \end{array}$$

Ex:



$$Adj_D = \begin{array}{cc} \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \\ & \begin{matrix} a & b & c & d \end{matrix} \end{array}$$

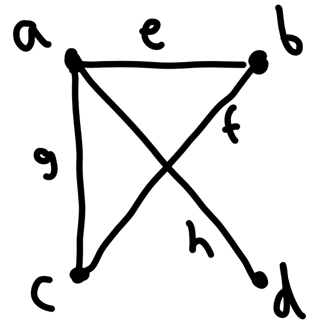
Def: Let G be a graph w/ vertices $v_1, \dots, v_n$.
and edges $e_1, \dots, e_m$

The incidence matrix of G is the matrix $\mathrm{Inc}_G = [m_{ij}]$

or both endpoints!

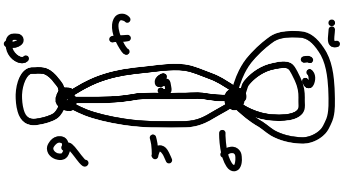
where $m_{ij} = \begin{cases} 1, & \text{if } v_i \text{ is an endpoint of } e_j \\ 0, & \text{otherwise} \end{cases}$

Ex:

G: 

$$\mathrm{Inc}_G = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} e & f & g & h \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Ex:

G: 

$$\mathrm{Inc}_G = \begin{array}{c} \\ a \\ b \end{array} \begin{array}{c} \begin{array}{cccccc} e & f & g & h & i & j \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

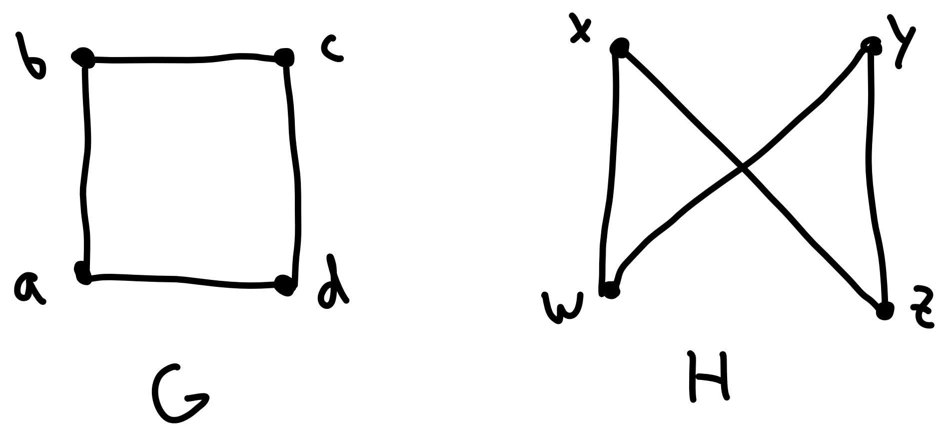
Def: Let $G = (V_1, E_1)$ and $H = (V_2, E_2)$ be simple graphs. A function $f: V_1 \to V_2$ is an <u>isomorphism</u> if

a) f is a bijection

b) $f(a)$ and $f(b)$ are adj. if and only if a and b are adj.

If any isomorphism exists, G and H are <u>isomorphic</u>

Ex 8:



Graph isomorphism:

$f(a) = w \qquad f(c) = z$

$f(b) = x \qquad f(d) = y$

Isomorphic graphs have to have the same:

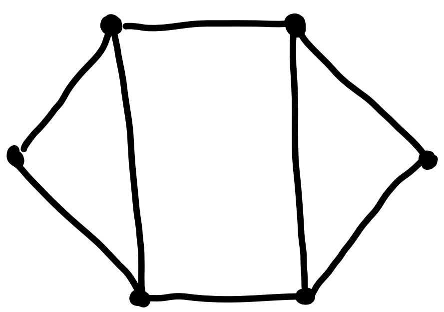
a) number of vertices

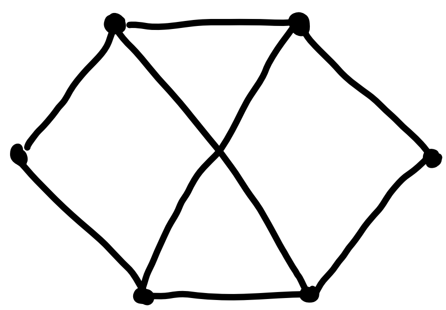
b) number of edges

c) lists of degrees

So if G & H differ on any of these ⇝ not isomorphic!

Be careful:





same a), b), c), but not isomorphic!

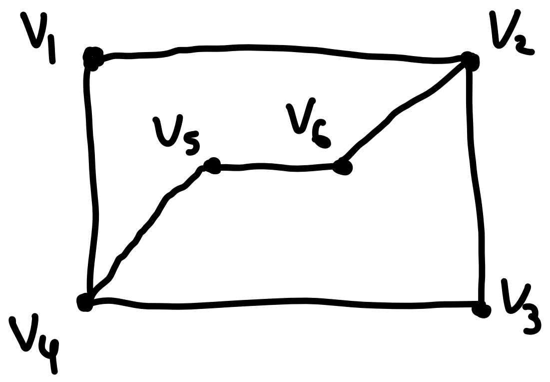
Two ways to show two graphs <u>are</u> isom.

1) Find an isomorphism

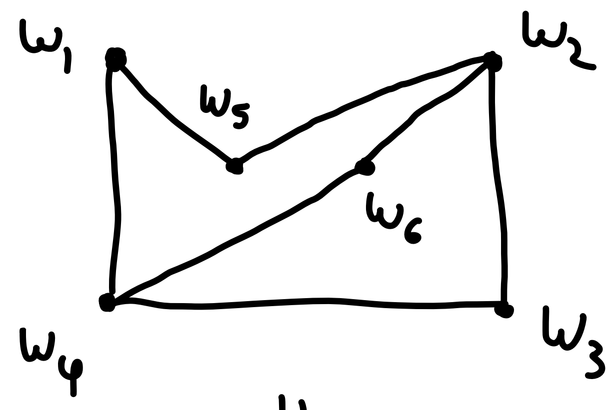
2) Show that the adjacency matrices are the same for some ordering of the vertices

(always same ordering on rows & cols!)

Ex 11:



G



H

$$\mathrm{Adj}_G = \begin{array}{cc} & \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\mathrm{Adj}_H = \begin{array}{cc} & \begin{array}{cccccc} w_1 & w_2 & w_3 & w_4 & w_5 & w_6 \end{array} \\ \begin{array}{c} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Not the same

But... put the vertices in a different order, and

$$\begin{array}{c|cccccc|}
 & w_6 & w_2 & w_3 & w_4 & w_1 & w_5 \\
\hline
w_6 & 0 & 1 & 0 & 1 & 0 & 0 \\
w_2 & 1 & 0 & 1 & 0 & 0 & 1 \\
w_3 & 0 & 1 & 0 & 1 & 0 & 0 \\
w_4 & 1 & 0 & 1 & 0 & 0 & 0 \\
w_1 & 0 & 0 & 0 & 1 & 0 & 1 \\
w_5 & 0 & 1 & 0 & 0 & 1 & 0 \\
\hline
\end{array}$$

So G and H are isomorphic.