## Announcements:

HW1 due Sunday 11:59pm via Gradescope

Talk to me now if you have Gradescope issues

Quiz 1 on Wed. (def'ns, basic facts)

Lecture 2 notes + video posted

- Venn diagrams
- Subsets
- Power set
- Cartesian product
- Cardinality
- Union
- Intersection
- Set-minus
- Complement

LaTeX tutorial video posted

Today: set identities and proof techniques

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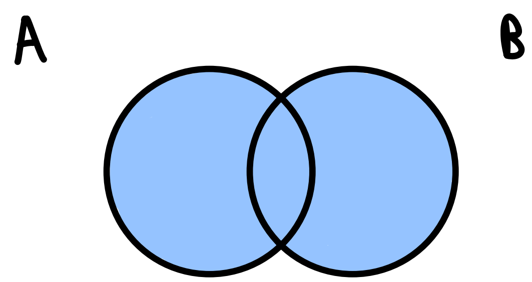
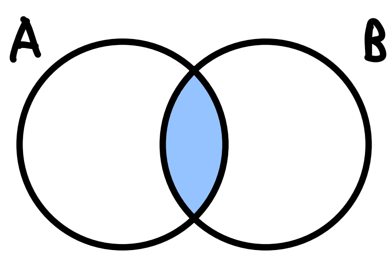
Recall:

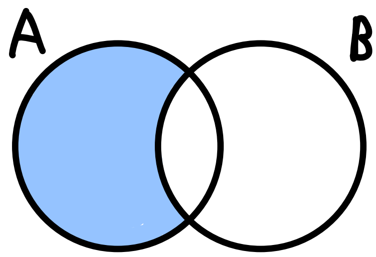
$A \cup B$



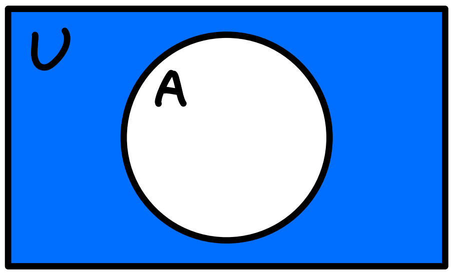

$A \cap B$



$A \setminus B$




$\overline{A}$



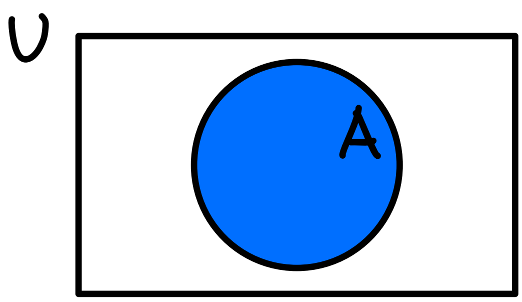

## Set identities

Let $A, B, C$ be sets, and let $U$ be the universal set (always have $A, B, C \subseteq U$)

1) Identity laws

$A \cap U = A$

$A \cup \phi = A$




2) Domination laws

$A \cup U = U$

$A \cap \phi = \phi$

3) Idempotent laws

$A \cup A = A$

$A \cap A = A$

4) Complementation law

$$\overline{\overline{A}} = A$$

5) Commutative laws

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

6) Associative laws

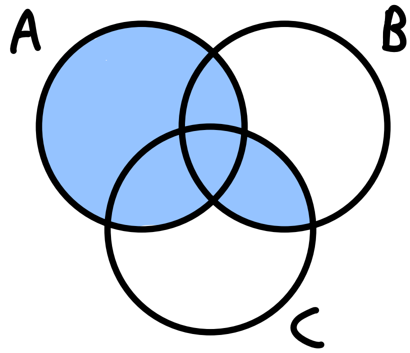
$$A \cup (B \cup C) = (A \cup B) \cup C$$

$$A \cap (B \cap C) = (A \cap B) \cap C$$

7) Distributive laws

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$




8) de Morgan's Laws

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

9) Absorbtion laws

$$A \cup (A \cap B) = A$$

$$A \cap (A \cup B) = A$$

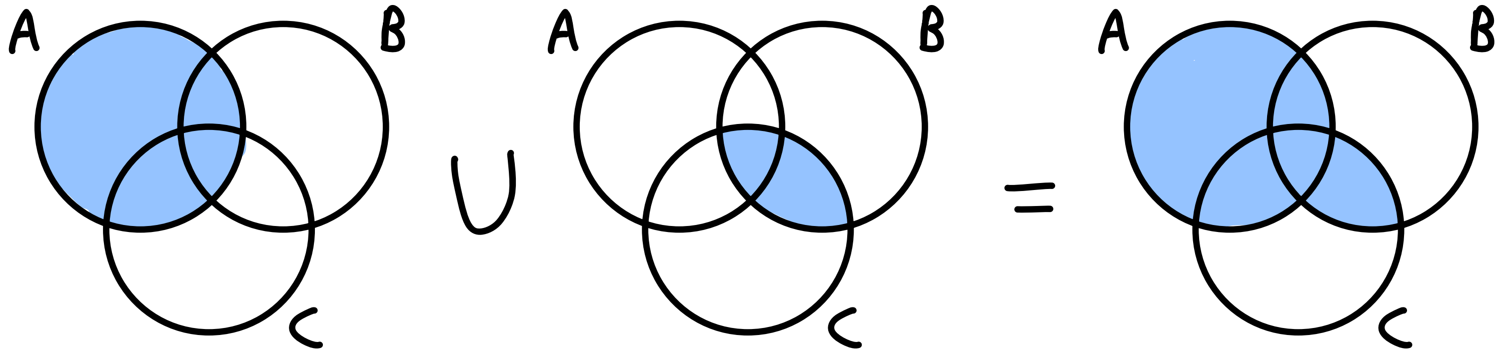
10) Complement laws

$$A \cup \overline{A} = U$$

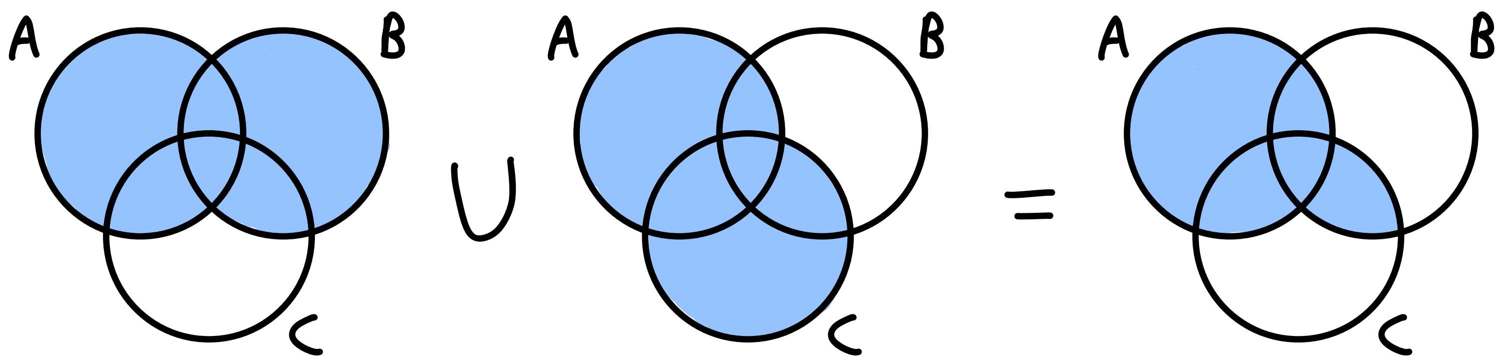
$$A \cap \overline{A} = \emptyset$$

# Venn diagram tricks (for intuition only)

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$




$A$ $\quad$ $B \cap C$ $\quad$ $A \cup (B \cap C)$




$A \cup B$ $\quad$ $A \cup C$ $\quad$ $(A \cup B) \cap (A \cup C)$

# Proof Techniques

A proof is an argument that is

- precise (say exactly what you mean)
- rigorous (justify each step)
- complete (no logical holes)
- clear (easy to read / understand)

Q: Shouldn't all solutions have these properties?

A: Yes, but we'll have particularly high standards on proofs
and you should always show your work!

Problems which say "prove", "show", "demonstrate" require proof

On HW1, this is

2.1.26, 2.2.15, 2.2.24

If in doubt, ask!

Examples of good proofs: §2.2 Examples 10, 11, 12, 13, 14

Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Pf: We show that $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ and $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$

<u>$A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$:</u>

Let $x \in A \cup (B \cap C)$. Then $x \in A$ or $x \in B \cap C$ (or both). In the first case, $x \in A \cup B$ since $x \in A$, and $x \in A \cup C$ since $x \in A$. Therefore $x \in (A \cup B) \cap (A \cup C)$. In the second case, $x \in B$ and $x \in C$, so $x \in A \cup B$ and $x \in A \cup C$, so $x \in (A \cup B) \cap (A \cup C)$. Hence, $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

<u>$(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$</u>

Let $x \in (A \cup B) \cap (A \cup C)$. If $x \in A$, then $x \in A \cup (B \cap C)$. If $x \notin A$, then since $x \in A \cup B$, $x \in B$, and since $x \in A \cup C$, $x \in C$. Thus, $x \in B \cap C$, so $x \in A \cup (B \cap C)$. Hence, $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$

end of proof $\longrightarrow \square$

Another method: use membership tables.

Every elt. $x$ has 8 possibilities:

$x \in A$, $x \in B$, $x \in C$

$x \in A$, $x \in B$, $x \notin C$,

etc.

| A | B | C |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 0 |

1 means $x \in$ set

0 means $x \notin$ set

Pf (alt strategy):

We draw the membership table for both sides of the desired equality. Since the columns for $A \cup (B \cap C)$ and $(A \cup B) \cap (A \cup C)$ are identical, the sets are equal.

| $A$ | $B$ | $C$ | $B \cap C$ | $A \cup (B \cap C)$ | $A \cup B$ | $A \cup C$ | $(A \cup B) \cap (A \cup C)$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$\square$