Announcements

Midterm 2: Friday in class

Covers through Chapter 8

Reference sheet allowed (one A4 sheet w/ writing on both sides)

See policy email (practice problems, etc.)

Thurs. problem/review session (probably) moving online (thoughts?)

§9.1: Relations

Def: Let A and B be sets. A relation from A to B is a subset R of $A \times B$.

Write aRb to mean $(a,b) \in R$

Ex: $A = \{a, b, c, d, e\}$ students

$B = \{C1, C2, DM\}$ math classes

$R = \{(a, C1), (a, DM), (b, DM), (c, C2), (c, DM), (e, C1), (e, C2), (e, DM)\}$

a is taking C1 & DM

b is taking DM

c is taking C2 & DM

d is taking nothing

e is taking C_1, C_2, & DM

aRC_1, aRC_2, etc.

R is a function if every elt. of A appears exactly once in R.

Often, we care about relations from A to A ("on A")

Class activity: Let $A = \{1,2,3\}$

Match the symbols w/ the relations

i) $\leq$ ii) $>$ iii) $=$ iv) $|$ ("divides")

a) $\{(1,1), (1,2), (1,3), (2,2), (3,3)\}$

b) $\{(1,1), (2,2), (3,3)\}$

c) $\{(2,1), (3,1), (3,2)\}$

d) $\{(1,1), (1,2), (1,3), (2,2), (2,3), (3,3)\}$

Ex 6: If $|A| = n$, there are 2^{n^2} relations on A, n^n of which are functions.

e.g. $n = 3$: 512 relations, 27 functions

$n = 4$: 65536 relations, 256 functions

Properties:

- R is reflexive if aRa for all $a \in A$
- R is symmetric if whenever aRb, then bRa
- R is antisymmetric if whenever aRb and $a \neq b$, then $b \not R a$
- R is transitive if whenever aRb and bRc, then aRc

Class activity: For each of these relations on $\mathbb{Z}$, determine whether it has each of the above properties:

R_1: $\leq$

R_2: $>$

R_3: $=$

R_4: $|$

R_5: $\{(a,b) \mid a = b \text{ or } a = -b\}$

R_6: $\{(a,b) \mid a = b+1\}$

R_7: $\{(a,b) \mid a+b \leq 3\}$

R_8: $\{(1,2), (2,3), (3,1)\}$

R_9: $\{(1,2), (2,3), (1,3)\}$

R_{10}: $\{(1,1), (2,2), (3,3)\}$

R_{11}: $\{(1,5), (5,1), (2,2), (2,3), (3,2)\}$

$R_{12} = R_3 \cup R_8$

$R_{13} = R_3 \cap R_8$

union/intersection as sets

Operations:

- Complement: $\overline{R} = \{(a,b) \in A \times B \mid (a,b) \notin R\}$
- Inverse: $R^{-1} = \{(b,a) \mid (a,b) \in R\}$ relation from B to A
- Composition: If $R \subseteq A \times B$, $S \subseteq B \times C$, then

  $S \circ R = \{(a,c) \in A \times C \mid$ there exists $b \in B$ s.t. $(a,b) \in R, (b,c) \in S\}$
  relation from A to C

Ex: $A = \{1, 2, 3\}$, $B = \{(1,2,3,4)\}$

$R = \{(1,1), (1,2), (2,1), (2,2), (3,3), (3,4)\}$

Class activity: find

$\overline{R}$, R^{-1}, $R \circ R^{-1}$, and $R^{-1} \circ R$