Announcements

HW1 posted, due @ 11:59pm Sunday via Gradescope

Problem sessions scheduled: Thurs. 10-11:30am Everitt Lab. 2101

$A = \{x \mid x \text{ is an odd integer}\}$

$= \{\ldots, -5, -3, -1, 1, 3, 5, 7, \ldots\}$

$B = \{x \in \mathbb{Z} \mid -2 \leq x \leq 3\} = \{-2, -1, 0, 1, 2, 3\}$

Class activity: List all elements of the following sets

$C = \{x \in \mathbb{Z} \mid x \notin A \text{ and } x \in B\}$

$D = \{x \mid x \in \emptyset\}$

$E = \{A, \emptyset, \{\pi, e\}\}$

$C = \{-2, 0, 2\}$

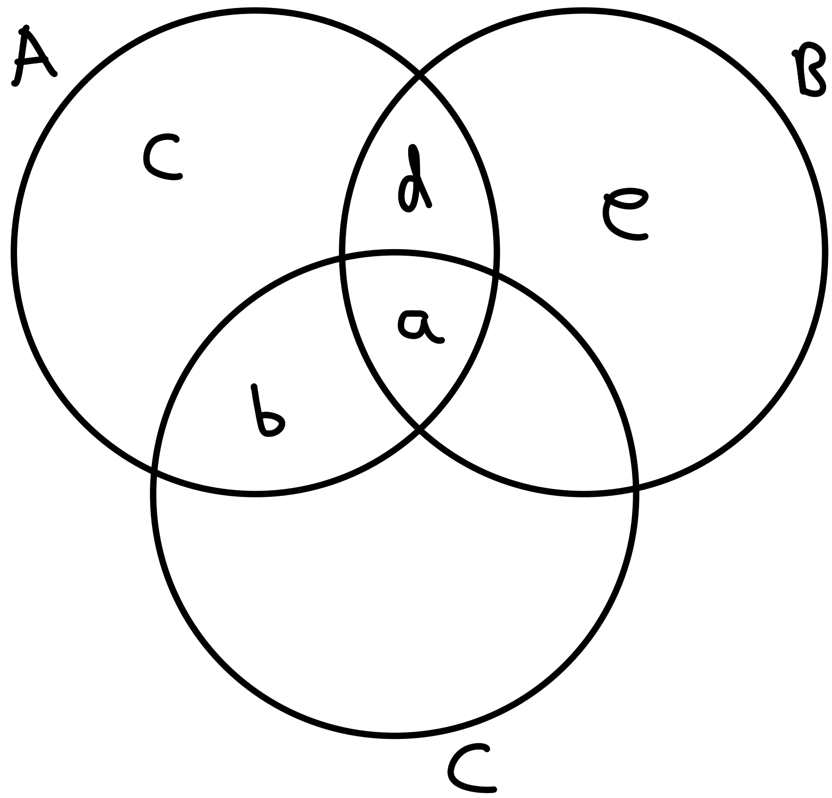
$D = \emptyset = \{\}$ (no elements)

Elements of E:

$\{\ldots, -5, -3, -1, 1, 3, \ldots\}, \ \emptyset, \ \{\pi, e\}$

Venn diagrams:

$A = \{a, b, c, d\}$ $B = \{a, d, e\}$ $C = \{a, b\}$



$C \subseteq A$

$B \not\subseteq A$

$A \not\subseteq B$

$A \not\subseteq C$

$C \not\subseteq B$

$B \not\subseteq C$

Def: A is a subset of B (write $A \subseteq B$) if every elt. of A is an elt. of B.

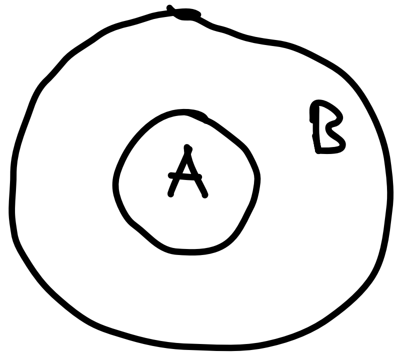
Always: $A \subseteq A$

$\emptyset \subseteq A$

$A \subseteq B$ and $B \subseteq A$ if and only if $A = B$

If $A \subseteq B$ but $B \nsubseteq A$, then A is a proper subset of B and we write

$$A \subsetneq B$$



$A \subseteq B$

Power set: $\mathcal{P}(A)$ is the set of all subsets of A

$$\mathcal{P}(A) = \{B \mid B \subseteq A\}$$

e.g. $A = \{1,2\}$

$$\mathcal{P}(A) = \{\phi, \{1\}, \{2\}, \underbrace{\{1,2\}}_{A}\}$$

Cartesian product: $A \times B$ is the set of all ordered pairs (a,b) where $a \in A$, $b \in B$

$A \times B = \{(a,b) \mid a \in A, b \in B\}$

e.g. $A = \{1,2\}$ $B = \{1,3\}$

$A \times B = \{(1,1), (1,3), (2,1), (2,3)\}$

Same for larger products; here we use ordered <u>tuples</u>

e.g. $(1,2,3,4,5)$ is a 5-tuple

$A \times B \times C \times D = \{(a,b,c,d) \mid a \in A, b \in B, c \in C, d \in D\}$

Cardinality: the size of a set (num. elts.)

Cardinality of A: $|A|$

e.g. $\left.\begin{array}{l} |\emptyset| = 0 \\ |\{1,2,3\}| = 3 \end{array}\right\}$ finite cardinality

$|\mathbb{Z}| = |\mathbb{R}| = |\mathbb{N}| = \infty \}$ infinite cardinality

Class activity:

a) What is $|P(A)|$ in terms of $|A|$?

b) What is $|A \times B|$ in terms of $|A|$ and $|B|$?

e.g. $A = \{1,2,3\}$ $\quad$ $B = \{1,2\}$

$|A| = 3$ $\quad$ $|B| = 2$

$$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$$

$|P(A)| = 8$

$$|P(A)| = 2^{|A|}$$

$|\phi| = 0 \quad P(\phi) = \{\phi\} \quad |P(\phi)| = 1 = 2^0$

§2.2 : Set operations

Union:

$$A \cup B = \{x \mid x \in A \text{ or } x \in B \text{ (or both)}\}$$

Intersection:

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

Set-minus:

$$A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$$

(book uses $A - B$)

e.g. $A = \{1,2,3,4\}$ $\quad B = \{1,3,5,7\}$

$A \cup B = \{1,2,3,4,5,7\}$

$A \cap B = \{1,3\}$

$A \setminus B = \{2,4\}$

$B \setminus A = \{5,7\}$

Complement: Fix a "universal" set U. Then,

$$\overline{A} = U \setminus A.$$

e.g. $U = \mathbb{Z}$

$A = \{x \in \mathbb{Z} \mid x \text{ is odd}\}$

$\overline{A} = \{x \in \mathbb{Z} \mid x \text{ is even}\}$

Friday: We'll discuss set identies like

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

See also LaTeX tutorial