Announcements

HW6 posted (due Sun. 10/13)

Quiz 4 Wed. in-class

Midterm feedback:

- Lecture pace seems fine
- Counting is either favorite or least favorite
- Homework good level and/or difficult
- Proofs: sometimes diff. to know expectation
- Office hour times
- Homework should be posted earlier, and not have so many q's from Friday lecture!

§7.2: Probability Theory

Recall: every outcome has a probability $p(s)$

$0 \leq p(s) \leq 1$ for all $s \in S$

$$\sum_{s \in S} p(s) = 1$$

add up $p(s)$ for every elt. $s \in S$

If E is an event,

$$p(E) = \sum_{s \in E} p(s)$$

Recall the complement $\overline{E} = S \setminus E$ of E

$E_1 \cup E_2$: either is true

$E_1 \cap E_2$: both true

1) Complement rule: $p(\overline{E}) = 1 - p(E)$
2) Subtraction rule: $p(E \cup F) = p(E) + p(F) - p(E \cap F)$
3) Sum rule: if E & F disjoint, $p(E \cup F) = p(E) + p(F)$

Ex: 3 coins

$|S| = 8$

E : at least one head

F : at least one tail

$\bar{E}$: no heads

$\bar{F}$: no tails

$p(\bar{E}) = p(\bar{F}) = \frac{1}{8}$

so $p(E) = p(F) = 1 - \frac{1}{8} = \frac{7}{8}$

$p(\bar{E} \cup \bar{F}) = p(\bar{E}) + p(\bar{F}) - p(\bar{E} \cap \bar{F}) = \frac{1}{8} + \frac{1}{8} - 0 = \frac{2}{8}$

By de Morgan's Laws, $\overline{E \cup F} = \bar{E} \cap \bar{F}$

So $p(E \cup F) = 1 - p(\overline{E \cup F}) = 1 - p(\bar{E} \cap \bar{F}) = 1$

$p(E \cap F) = p(E) + p(F) - p(E \cup F) = \frac{7}{8} + \frac{7}{8} - 1 = \frac{6}{8}$

Let E, F be events, $p(F) > 0$. The <u>conditional probability</u> of E given F is

$$p(E|F) := \frac{p(E \cap F)}{p(F)}$$

Basic idea: If we <u>know</u> F is true, what is the chance E is true

Ex (cont.):

$p(E|F) = \frac{6/8}{7/8} = \frac{6}{7}$ $\qquad$ $p(F|E) =$ same

$p(E|\overline{E}) = \frac{0}{1/8} = 0$

$p(E|\overline{F}) = \frac{p(E \cap \overline{F})}{p(\overline{F})} = \frac{?}{7/8} = 1$

If $\overline{F}$ true,
E always true

Independence: E and F are <u>independent</u> if and only if

$$p(E \cap F) = p(E)\,p(F)$$

$E_1, \ldots, E_n$ are pairwise independent if for all i, j,

$$p(E_i \cap E_j) = p(E_i)\,p(E_j)$$

$E_1, \ldots, E_n$ are (mutually) independent if every eqn

$$p(E_{i_1} \cap \cdots \cap E_{i_k}) = p(E_{i_1}) \cdots p(E_{i_k})$$

holds.

e.g. $p(E_1 \cap E_4 \cap E_6) = p(E_1)\,p(E_4)\,p(E_6)$

Ex (cont): $p(E \cap F) = \frac{6}{8}$

not equal, so not indep.

$p(E)\,p(F) = \frac{7}{8} \cdot \frac{7}{8} = \frac{49}{64}$

Bernoulli trials: successive independent weighted coin flips.

$p(\text{success}) = p$

$p(\text{failure}) = q = 1-p$

Think: coin flips w/ $p(H) = p$

Class activity: Flip 3 coins w/ $p(H) = \frac{2}{3}$. Find the prob of

a) no H

b) exactly one H

c) exactly two H's

d) three H's

General formula: n Bernoulli trials. The probability of exactly k successes is:

$$\binom{n}{k} p^k q^{n-k}$$

$\binom{n}{k}$ orders

k successes

$n-k$ failures

Also called the <u>binomial distribution</u>

Ex 9: Generate a binary string where each digit is generated independently and has a 0.9 chance of being a 0. What is the probability that the string has exactly 8 0's?

Ans:

$$p(\text{exactly } 8 \text{ } 0\text{'s}) = \binom{10}{8} 0.9^8 \cdot 0.1^2 = 0.1937$$