Announcements

Midterm 1 Wed in-class (50 minutes)

- Reference sheet allowed (one A4 sheet, both sides)
- No other resources
- Sections covered: 2.1-3, 3.1-2, 5.1-2, 6.1-2
- Problem session next week → review session on Tues. (time/location TBD)
- Practice problems posted

See policy email for more

Subtraction rule: If a task can be done either in one of m ways or one of n ways, with overlap of k, then there are $m+n-k$ ways to do the task.

$$|A \cup B| = |A| + |B| - |A \cap B|$$
$$\qquad\quad m \quad n \quad k$$

Ex 18: How many 01-strings of length 8 either start w/ 1 or end w/ 00?

Start w/ 1:

$1 * * * * * * *$

$1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 128$ choices

End w/ 00

$*******00$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 1 \cdot 1 = 64$ choices

Start w/ 1 AND end w/ 00:

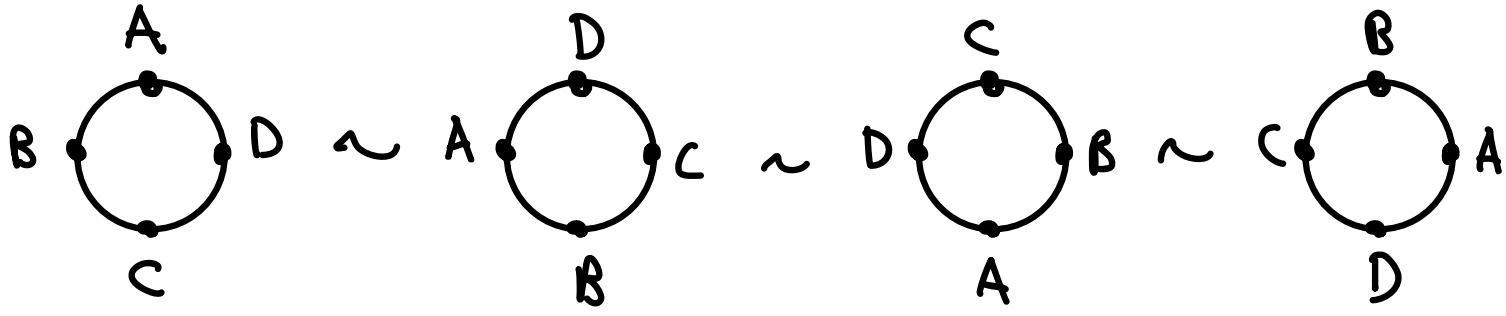
$1******00$

$1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 1 \cdot 1 = 32$ choices

Ans: $128 + 64 - 32 = 160$ strings

Division rule: If there are n ways to do a task, and groups of d of these ways are equivalent, then there are n/d ways up to equivalence.

Ex 20: How many different ways are there to seat 4 people around a circular table, where two seatings are considered equivalent if they are rotations of each other?



4 rotations of each seating arrangement

$4 \cdot 3 \cdot 2 \cdot 1 = 24$ seating arrangements

$\frac{24}{4} = 6$ nonequivalent seating arrangements

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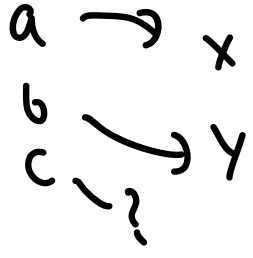
§6.2 : The Pigeonhole Principle

Pigeonhole Principle: Put m pigeons into n boxes. If $m > n$, there must be at least one box w/ multiple pigeons.

Ex:

a) If $f: A \to B$ and $|A| > |B|$, then f is not 1-1.



b) Among any group of 367 people, there must be at least two who share a birthday

c) For every positive integer n, there is a (nonzero) multiple of n whose base-10 expansion has just 0's and 1's.

Eg. $n=4$, $4 \cdot 25 = 100$

Class activity: Find such a multiple of 6

Pf: Consider the $n+1$ integers

$$a_1 = 1$$
$$a_2 = 11$$
$$a_3 = 111$$
$$\vdots$$
$$a_{n+1} = \underbrace{11\cdots1}_{n+1\ 1\text{'s}}$$

Divide each a_i by n, and let r_i be the remainder. Each r_i is an integer from 0 to $n-1$, so by the pigeonhole principle there exist $i<j$ s.t. $r_i = r_j$. Then $n \mid a_j - a_i$ and $a_j - a_i$ has

decimal expansion $\underbrace{1\cdots1}_{j-i}\underbrace{0\cdots0}_{i}$. □

e.g. $n=6$

$a_1=1$ $\quad r_1=1$

$a_2=11$ $\quad r_2=5$

$a_3=111$ $\quad r_3=3$

$a_4=1111$ $\quad r_4=1$

$a_5=11111$ $\quad r_5=5$

$a_6=111111$ $\quad r_6=3$

$a_7=1111111$ $\quad r_7=1$

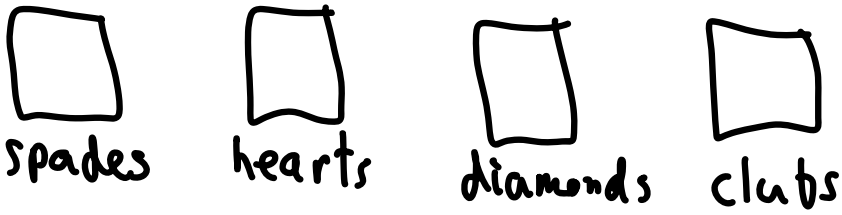
$1111-1=1110=185\cdot6.$

Generalized pigeonhole principle: Put m pigeons into n boxes. Then there is at least one box w/ $\lceil m/n \rceil$ pigeons

e.g. $m=31 \quad n=10 \rightsquigarrow \lceil m/n \rceil = \lceil 3.1 \rceil = 4$

$m=40 \quad n=10 \rightsquigarrow \lceil m/n \rceil = \lceil 4 \rceil = 4$

Ex 7: How many cards must be chosen from a deck to ensure there are ≥ 3 of the same suit



Ans: We want the smallest m s.t.

$\left\lceil \frac{m}{4} \right\rceil \geq 3$ i.e. $m > 2 \cdot 4 \rightsquigarrow m = 9$

Ex 8: Telephone numbers are of the form,

$\underbrace{NXX}_{\text{area code}}$-NXX-XXXX

where each N can be a digit from 2 to 9 and each X can be a digit from 0 to 9.
A state has 25,000,000 phones. How many area codes does it need to ensure each phone has a diff. num?

Soln: NXX-XXXX

$8 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 8{,}000{,}000$ numbers per area code

$m = 25$ million, $n = 8$ million

$\lceil m/n \rceil = \lceil \frac{25}{8} \rceil = 4$

So we need 4 area codes.